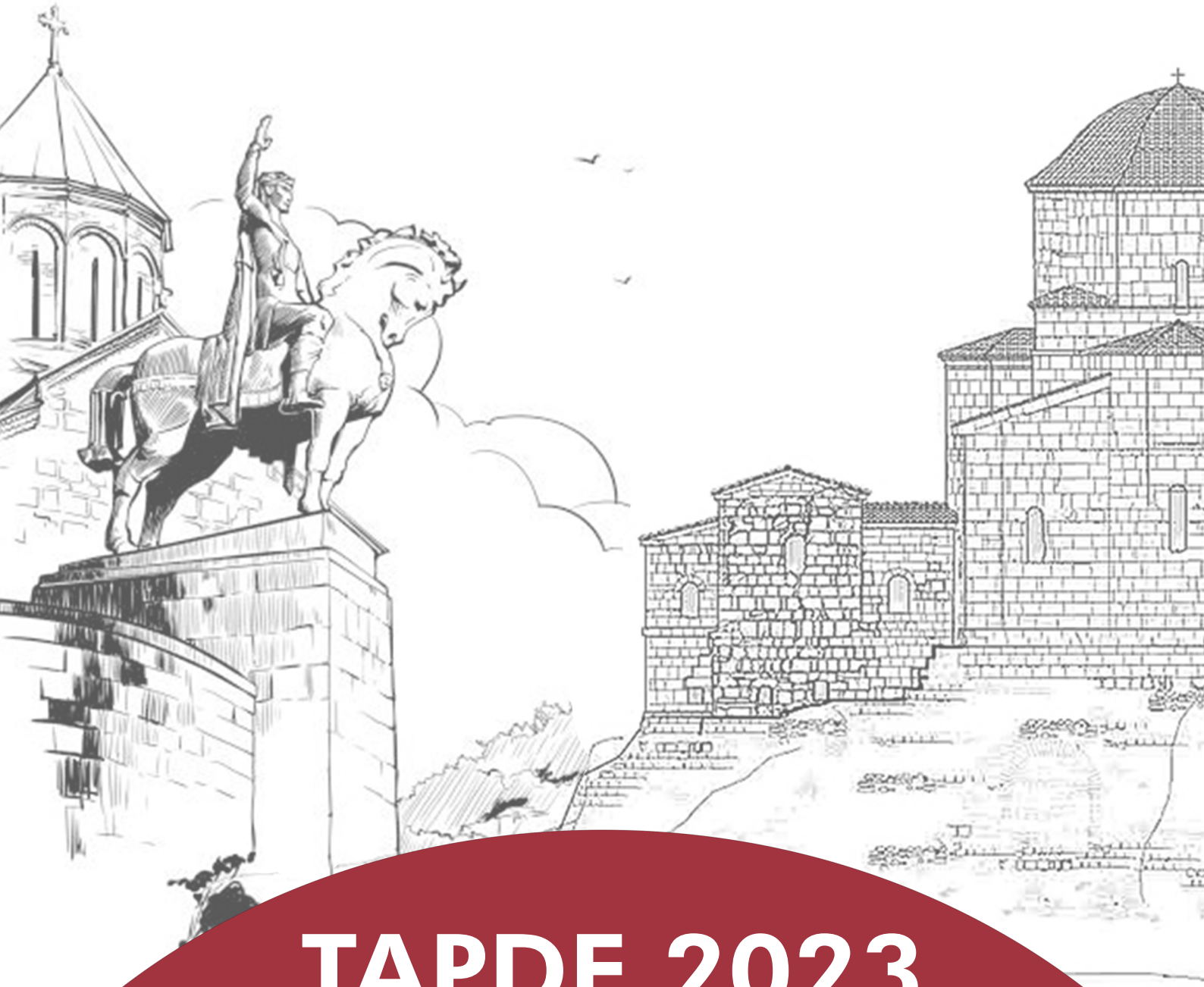

TAPDE 2023

PROGRAM & ABSTRACTS

THE TBILISI ANALYSIS & PDE WORKSHOP

AUGUST 30 - SEPTEMBER 02, 2023

The University of Georgia, Tbilisi

The book of abstracts has been assembled by *Zurab Vashakidze*
(**e-mail:** z.vashakidze@ug.edu.ge).

The digital edition of this booklet is accessible through the following link:
https://tapde-workshop.ug.edu.ge/

The source of this template can be traced back to LaTeXTemplates.com, and it has been developed based on the initial version accessible at:
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The most recent update pertaining to both the **Program** and the **Book of Abstracts** was completed on **Aug. 28, 2023**.

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About

This is a generic version of the real AMCOS conference booklet for which this LaTeX template was generated. All information about the use and distribution of this template, and all related codes, can be found at `https://github.com/maximelucas/AMCOS_booklet`.

TAPDE2023

The Tbilisi Analysis & PDE Seminar (TAPDE) was founded in November 2020 as a bi-weekly series of online seminars, featuring presentations by over 40 distinguished scholars from several countries. The original plan to complement the online talks with an annual Workshop had to be put on hold due to COVID-19. We are delighted to be able to announce the launch of the Tbilisi Analysis & PDE Workshop 2023 (TAPDE2023), which will take place in Tbilisi, the capital of Georgia.

The workshop encompasses a wide variety of topics, such as real and complex analysis, operator theory, harmonic analysis, integral equations, numerical analysis, partial differential equations, problems of mathematical physics, and related fields. We welcome submissions from researchers interested in presenting their latest research findings, sharing their expertise, and discussing new developments in their respective fields.

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Timetable

IS: Invited Speaker, CT: Contributed Talk.

August 30, Wednesday, Session at the University of Georgia, Tbilisi

08:30–09:00	**Registration of participants, Room 519**		
09:00–09:20	**Opening ceremony, Room 519**		
Room 519	**Plenary Session**		
Chairman	**Roland Duduchava**		
09:20–10:10	IS	**Pavel Exner** Prague, Czechia	Curvature-Induced Bound States in Soft Quantum Waveguides
10:20–11:10	IS	**Sergey E. Mikhailov** London, United Kingdom	About Evolution Anisotropic Periodic Variable-Coefficient Navier-Stokes Equations
11:20–11:50	**Coffee break**		
Chairman	**Oleksiy Karlovych**		
11:50–12:40	IS	**Jani Virtanen** Reading, United Kingdom & Helsinki, Finland	Asymptotics of Block Toeplitz Determinants with Piecewise Continuous Functions
12:40–15:00	**Lunch**		
Room 519	**Semi-Plenary Session**		
Chairman	**Sergey E. Mikhailov**		
15:00–15:50	IS	**David Natroshvili** Tbilisi, Georgia	An Alternative Potential Method for Mixed Boundary Value Problems
Room 710	**Semi-Plenary Session**		
Chairman	**Yuri Karlovich**		
15:00–15:50	IS	**George Tephnadze** Tbilisi, Georgia	Almost Everywhere Convergence of Partial Sums and Certain Summability Methods of Trigonometric and Vilenkin Systems
Room 519	**Section of Differential Equations**		
Chairman	**David Natroshvili**		
16:00–16:25	CT	**George Chkadua** Tbilisi, Georgia	Mathematical Problems of Dynamical Interaction of Fluids and Multiferroic Solids
16:30–16:55	CT	**Otar Chkadua** Tbilisi, Georgia	Solvability, Asymptotic Analysis and Regularity Results of Dynamical Interface Crack Type Transmission Problems of the Generalized Thermo-Electro-Magneto-Elasticity for Composed Structures
17:00–17:30	**Coffee break**		
17:30–17:55	CT	**Lasha Ephremidze** Abu Dhabi, United Arab Emirates	On Spectral Factorization of Rational Matrix Functions
Room 710	**Section of Analysis**		
Chairman	**George Tephnadze**		
16:00–16:25	CT	**Santeri Miihkinen** Reading, United Kingdom	The Infinite Hilbert Matrix on Spaces of Analytic Functions
16:30–16:55	CT	**Gvantsa Shavardenidze** Tbilisi, Georgia	On the Convergence of Cesàro Means of Negative Order of Vilenkin-Fourier Series
17:00–17:30	**Coffee break**		
17:30–17:55	CT	**Giorgi Tutberidze** Tbilisi, Georgia	Properties of the Sequence of Linear Functionals on BV with Application

18:15	**Wine Reception hosted by the President of the University of Georgia, Professor Giuli Alasania**

August 31, Thursday, Session at Sevsamora, Saguramo

09:00	**Departure from the front door of the University of Georgia, Tbilisi**		
Conference Room	**Plenary Session**		
Chairman	**Pavel Exner**		
10:00–10:50	IS	**Oleksiy Karlovych** Caparica, Portugal	Discrete Riesz Transforms on Rearrangement Invariant Banach Sequence Spaces and Maximally Noncompact Operators
11:00–11:50	IS	**Guillermo P. Curbera** Sevilla, Spain	The Finite Hilbert Transform Acting on the Zygmund Space $L\log L$
12:00–12:30	**Coffee break**		
Chairman	**Elijah Liflyand**		
12:30–13:20	IS	**Diogo Oliveira e Silva** Lisboa, Portugal	Sharp Restriction Theory: Highlights and Future Directions
13:30–15:00	**Lunch**		
Conference Room	**Plenary Session**		
Chairman	**Guillermo P. Curbera**		
15:00–15:50	IS	**Elijah Liflyand** Ramat-Gan, Israel	L^1 Convergence of Fourier Transforms
Conference Room	**Section of Differential Equations**		
Chairman	**Eugene Shargorodsky**		
16:00–16:25	CT	**Joachim Gwinner** Neubiberg (Munich), Bavaria, Germany	On Frictional Unilateral Contact in Finite Strain Elasticity
16:30–16:55	CT	**Yuri Karlovich** Cuernavaca, México	Banach Algebras of Convolution Type Operators with PQC Data
17:00	**Departure to Mtskheta or Tbilisi (optional)**		
19:00	**Departure to Conference dinner at Restaurant Tsiskvili. Transport will depart from Mtskheta and from Tbilisi (University of Georgia)**		

The conference dinner will be held at the restaurant **"Tsiskvili"**, located at **99 Beliashvili St., Tbilisi** (see Google Maps: https://goo.gl/maps/GSbnaLKAUDb34xTG9).

September 01, Friday, Day off, Excursion

09:00–19:00	**Excursion to Kakheti, Departure from the front door of the University of Georgia, Tbilisi**

September 02, Saturday, Session at the University of Georgia, Tbilisi

Room 519	**Plenary Session**		
Chairman	**Jani Virtanen**		
09:00–09:50	IS	**Duván Cardona (Online)** Ghent, Belgium	Regularity Properties of Fourier Integral Operators: Old and New

10:00–10:50	IS	**Michael Ruzhansky (Online)** Belgium & United Kingdom	Logarithmic Sobolev Inequalities
11:00–11:30	**Coffee break**		
Chairman	**Zurab Vashakidze**		
11:30–12:20	IS	**Vladimir Rabinovich** México City, México	Fredholm Properties of Interaction Problems Associated with $n-$dimensional Singular Dirac Operators
12:30–15:00	**Lunch**		
Room 519	**Semi-Plenary Session**		
Chairman	**Vladimir Rabinovich**		
15:00–15:50	IS	**Roland Duduchava** Tbilisi, Georgia	Convolutions on Lie Groups, Generic Bessel Potential Spaces and Applications
Room 710	**Semi-Plenary Session**		
Chairman	**Diogo Oliveira e Silva**		
15:00–15:50	IS	**Eugene Shargorodsky** London, United Kingdom	Hörmander's Theorem for Translation Invariant Operators on Banach Function Spaces
Room 519	**Section of Differential Equations**		
Chairman	**Joachim Gwinner**		
16:00–16:25	CT	**Besiki Tabatadze** Tbilisi, Georgia	Comparative Analysis of Approximate Solutions for a Numerical Solution of a Two-Dimensional Nonlinear Model
16:30–16:55	CT	**Zurab Vashakidze** Tbilisi, Georgia	On a Convergence of a Three-Layer Semi-Discrete Approach for Solving the Nonlinear Dynamic Kirchhoff-Type String Equation with Time-Varying Coefficients
17:00–17:30	**Coffee break**		
17:30–17:55	CT	**Hicham Arroussi** Finland & United Kingdom	Toeplitz Operators on Large Fock Spaces
Room 710	**Section of Analysis**		
Chairman	**Santeri Miihkinen**		
16:00–16:25	CT	**Davit Baramidze** Georgia & Norway	Maximal Operators of Partial Sums of Walsh-Fourier Series in the Martingale Hardy Spaces
16:30–16:55	CT	**Lasha Baramidze** Tbilisi, Georgia	Summability of Tkebuchava's Means of One and Two-Dimensional Trigonometric Fourier Series
17:00–17:30	**Coffee break**		
17:30–17:55	CT	**Nika Areshidze** Tbilisi, Georgia	Approximation by Nörlund Means with Respect to Walsh System in Lebesgue Spaces
18:00–18:25	CT	**Mark G. Lawrence** Astana, Kazakhstan	Real Analytic Bergman Spaces
18:30	**Closing ceremony, Room 519**		

List of Abstracts

Invited Speakers (IS)

Regularity Properties of Fourier Integral Operators: Old and New

Duván Cardona IS

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Let X and Y be two smooth manifolds of the same dimension. It was proved by Seeger, Sogge and Stein in [2] that the Fourier integral operators with real non-degenerate phase functions in the class $I^{\mu}_{1}(X,Y;\Lambda)$, $\mu \leq -(n-1)/2$, are bounded from H^1 to L^1. The sharpness of the order $-(n-1)/2$, for any elliptic operator was also proved in [2] and extended to other types of canonical relations in [1]. That the operators in the class $I^{\mu}_{1}(X,Y;\Lambda)$, $\mu \leq -(n-1)/2$, satisfy the weak (1,1) inequality was proved by Terence Tao [3]. In this talk, we discuss the weak (1,1) inequality for the order $-(n-1)/2$ and its sharpness for any elliptic Fourier integral operator, as well as its versions for canonical relations satisfying additional rank conditions, illustrating that a type of converse for the result by Tao [3] holds.

References

[1] M. Ruzhansky. On the sharpness of Seeger-Sogge-Stein orders. *Hokkaido Math. J.*, 28, no. 2, 357–362, 1999.
[2] A. Seeger, C. D. Sogge, and E. M. Stein. Regularity properties of Fourier integral operators. *Ann. of Math.*, 134(2):231–251, 1991.
[3] T. Tao. The weak-type $(1,1)$ of Fourier integral operators of order $-(n-1)/2$. *J. Aust. Math. Soc.*, 76(1):1–21, 2004.

Convolutions on Lie Groups, Generic Bessel Potential Spaces and Applications

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The purpose of the presentation is to discuss the convolution integro-differential equations on Lie groups. In this framework, we suggest to underline the role of adapted (Generic) Bessel potential spaces (GBPS) in the structure of the underlying Lie group. The definition of GBPS is based on generic differential operators from the Lie algebra of the Lie group. Such generic Bessel potential spaces are adapted better to the investigation of integro-differential (of pseudo-differential) operators on Lie groups.

Among examples, we will expose is "starring" star-shaped domains in $\mathbb{R}^n$ and their boundaries (which might contain angles and incoming and outgoing peaks). For such Lie groups we calculate the Haar measure, Fourier transform, its inverse, convolution operators and generic differential operators (generated by the vector fields from the corresponding Lie algebra), which are convolutions and are used to define GBPS.

Necessary and sufficient conditions (these conditions are different) for the unique solvability of convolution integro-differential equations in GBPS are indicated.

Then we will address shortly some applications and concentrate on one of them-boundary value problems (BVPs) for the Laplace-Beltrami equation on a hypersurface $\mathcal{C}$ with the Lipschitz boundary, containing a finite number of angular points (knots). The Dirichlet, Neumann and mixed-type BVPs are considered in a non-classical setting when solutions are sought in the GBPS with weight. By the localization, the problem is reduced to the investigation of Model Dirichlet, Neumann and mixed BVPs for the Laplace equation in planar angular domains in GBPS. Explicit criteria for the Fredholm property and the unique solvability of the initial BVPs are obtained and singularities of solutions at knots of the mentioned BVPs are precisely indicated.

The presentation is partially based on joint results with M. Ruzhansky, D. Cardona, A. Hendrickx and M. Tsaava.

Curvature-Induced Bound States in Soft Quantum Waveguides

Pavel Exner[1,2]

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The subject of the talk is Schrödinger operators with an attractive potential in the form of a channel of a fixed profile built along a smooth curve Γ in $\mathbb{R}^d$, $d = 2, 3$, in particular, the ways in which the geometry of the curve and the profile influence their discrete spectrum. The latter may be nonempty if Γ is infinite and not straight, but asymptotically straight in a suitable sense; we find sufficient conditions under which this happens. Furthermore, we prove that if Γ is a loop of a fixed length, the ground state is optimized by a circular shape. We also describe an analogous problem in which the soft channel is replaced by an array of potential wells. If time allows, we present a weak-coupling asymptotics in the situation when a straight channel suffers a one-sided deformation.

References

[1] P. Exner. *Spectral properties of soft quantum waveguides*. J. Phys. A: Math. Theor. **53** (2020), 355302. DOI: https://doi.org/10.1088/1751-8121/aba0f0.
[2] P. Exner, V. Lotereichik. *Optimization of the lowest eigenvalue of a soft quantum ring*. Lett. Math. Phys. **111** (2021), 28. DOI: https://doi.org/10.1007/s11005-021-01369-2.
[3] P. Exner, S. Vugalter. *Bound states in bent soft waveguides*. arXiv:2304.14776 [math.SP]. DOI: https://doi.org/10.48550/arXiv.2304.14776.
[4] P. Exner. *Geometry effects in quantum dot families*. arXiv:2305.12748 [math.SP]. DOI: https://doi.org/10.48550/arXiv.2305.12748.
[5] P. Exner, S. Kondej, V. Lotoreichik. *Bound states of weakly deformed soft waveguides*. arXiv:2211.0198 [math.SP]. DOI: https://doi.org/10.48550/arXiv.2211.0198.

Discrete Riesz Transforms on Rearrangement Invariant Banach Sequence Spaces and Maximally Noncompact Operators

Oleksiy Karlovych[1], ***Eugene Shargorodsky***[2]

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We say that an operator between Banach spaces is maximally noncompact if its operator norm coincides with its Hausdorff measure of noncompactness. We prove that a translation-invariant operator acting from a translation-invariant Banach sequence space $X(\mathbb{Z}^d)$ to a translation-invariant Banach sequence space $Y(\mathbb{Z}^d)$ is maximally noncompact whenever the target space $Y(\mathbb{Z}^d)$ satisfies mild additional conditions. As a consequence, we show that the discrete Riesz transforms R_j, $j = 1, \dots, d$ on rearrangement-invariant Banach sequence spaces with non-trivial Boyd indices are maximally noncompact. We also observe that the same results are valid for translation-invariant operators between translation-invariant Banach function spaces $X(\mathbb{R}^d)$ and $Y(\mathbb{R}^d)$.

L^1 Convergence of Fourier Transforms

Elijah Liflyand

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The problem of L^1 convergence of trigonometric series is generalized to the non-periodic case in a more or less final form. Roughly, it can now be said that any known condition which guarantees integrability of the Fourier transform of a function of bounded variation f also leads to the L^1 convergence of its partial integrals provided that a universal necessary and sufficient condition $\lim\limits_{|t|\to\infty} |f(t)|\ ln\,|t| = 0$ is added.

About Evolution Anisotropic Periodic Variable-Coefficient Navier-Stokes Equations

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We consider the evolution (time-dependent) anisotropic Navier-Stokes equations with variable space-periodic tensor viscosity coefficient in $\mathbb{R}^n$, $n \geq 2$. Employing the Galerkin algorithm, we prove the existence of a global weak spatially-periodic solution for the Navier-Stokes system in a periodic Sobolev space. The solution uniqueness and regularity in dimensions $n \in \{2, 3, 4\}$ are also discussed.

An Alternative Potential Method for Mixed Boundary Value Problems

David Natroshvili IS

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We consider a new alternative potential method to investigate a mixed boundary value problem (BVP) for the Lamé system of elasticity in the case of the three-dimensional bounded domain $\Omega \subset \mathbb{R}^3$, when the boundary surface $S = \partial\Omega$ is divided into two disjoint parts, S_D and S_N, where the Dirichlet and Neumann type boundary conditions are prescribed respectively for the displacement vector and the stress vector. Our approach is based on the potential method. We look for a solution to the mixed boundary value problem in the form of a linear combination of the single layer and double layer potentials with densities supported respectively on the Dirichlet and Neumann parts of the boundary. This approach reduces the mixed BVP under consideration to a system of pseudo-differential equations which contain neither extensions of the Dirichlet or Neumann data, nor the Steklov-Poincaré type operator. Moreover, the right-hand sides of the resulting pseudo-differential system are vectors coinciding with the Dirichlet and Neumann data of the problem under consideration. The corresponding pseudodifferential matrix operator is bounded and coercive in the appropriate L_2-based Bessel potential spaces. Consequently, the operator is invertible, which implies the unconditional unique solvability of the mixed BVP in the Sobolev space $[W_2^1(\Omega)]^3$ and representability of solutions in the form of a linear combination of the single layer and double layer potentials with densities supported respectively on the Dirichlet and Neumann parts of the boundary. Using a special structure of the obtained pseudo-differential matrix operator, it is also shown that the operator is invertible in the L_p-based Besov spaces with $\frac{4}{3} < p < 4$, which under appropriate boundary data implies C^α-Hölder continuity of the solution to the mixed BVP in the closed domain $\overline{\Omega}$ with $\alpha = \frac{1}{2} - \varepsilon$, where $\varepsilon > 0$ is an arbitrarily small number.

Sharp Restriction Theory: Highlights and Future Directions

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Sharp inequalities have a rich tradition in harmonic analysis, going back to the epoch-making works of Beckner and Lieb for the sharp Hausdorff–Young and Hardy Littlewood–Sobolev inequalities. Even though the history of sharp restriction theory is considerably shorter, it is moving at an incredible pace. In this talk, we survey selected highlights from the past decade, describe some of our own contributions, and pose a few open problems which lie at the interface of Euclidean harmonic analysis and dispersive PDE.

The Finite Hilbert Transform Acting on the Zygmund Space $L\,logL$

Guillermo P. Curbera[1], ***Susumu Okada***[2], ***Werner J. Ricker***[3] IS

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The finite Hilbert transform T is a singular integral operator which maps the Zygmund space $L\log L := L\log L(-1,1)$ continuously into $L^1 := L^1(-1,1)$. By extending the Parseval and Poincaré-Bertrand formulae to this setting, it is possible to establish an inversion result needed for solving the airfoil equation $T(f) = g$ whenever the data function

g lies in the range of T within L^1 (shown to contain $L(\log L)^2$). Until now this was only known for g belonging to the union of all L^p spaces with $p > 1$. It is established (due to a result of Stein) that T cannot be extended to any domain space beyond $L\log L$ whilst still taking its values in L^1, i.e., $T\colon L\log L \to L^1$ is optimally defined.

References
[1] G. P. Curbera, S. Okada, W. J. Ricker. *The finite Hilbert transform acting on the Zygmund space* $L\log L$. Ann. Sc. Norm. Super. Pisa Cl. Sci., to appear.

Fredholm Properties of Interaction Problems Associated with $n-$dimensional Singular Dirac Operators

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We consider the Dirac operators on $\mathbb{R}^n, n \geq 2$ with singular potentials

$$D_{\boldsymbol{A},\Phi,m,\Gamma\delta_\Sigma} = \mathfrak{D}_{\boldsymbol{A},\Phi,m} + \Gamma\delta_\Sigma\,,$$

where

$$\mathfrak{D}_{\boldsymbol{A},\Phi,m} = \sum_{j=1}^{n}\alpha_j\left(-i\partial_{x_j} + A_j\right) + \alpha_{n+1}m + \Phi I_N \tag{1}$$

is a Dirac operator on $\mathbb{R}^n$ with the variable magnetic and electrostatic potentials $A = (A_1, ..., A_n)$ and Φ, and the variable mass m. In formula (1) α_j are the $N \times N$ Dirac matrices, that is $\alpha_j\alpha_k + \alpha_k\alpha_j = 2\delta_{jk}\mathbb{I}_N$, $\mathbb{I}_N$ is the unit $N \times N$ matrix, $N = 2^{[(n+1)/2]}$, $\Gamma\delta_\Sigma$ is a singular delta-type potential supported on C^2-hypersurfaces $\Sigma \subset \mathbb{R}^n$ being the common boundary of the open sets $\Omega_\pm$. Let $H^1(\Omega^\pm, \mathbb{C}^N)$ be the Sobolev spaces of $N-$dimensional vector-valued distributions $\boldsymbol{u}$ on $\Omega^\pm$, and

$$H^1(\mathbb{R}^n\diagdown\Sigma, \mathbb{C}^N) = H^1(\Omega_+, \mathbb{C}^N) \oplus H^1(\Omega_-, \mathbb{C}^N)\,.$$

We consider in the talk the following problems:
1. Self-adjointnes of unbounded operator $\mathcal{D}_{A,m,\Phi}$ in $L^2(\mathbb{R}^n, \mathbb{C}^N)$ associated with the formal operator $D_{\boldsymbol{A},\Phi,m,\Gamma\delta_\Sigma}$.
2. Fredholm properties of the interaction (transmission) problems associated with $D_{\boldsymbol{A},\Phi,m,\Gamma\delta_\Sigma}$ on closed and unbounded C^2-hypersurfaces Σ.
3. Reduction of the interaction problems for the singular Dirac operators to integral equations on hypersurface Σ.

References
[1] V. Rabinovich. Dirac operators with delta-Interactions on smooth hypersurfaces in $\mathbb{R}^n$, Journal of Fourier Analysis and Applications (2022) 28:20.

Logarithmic Sobolev Inequalities

Michael Ruzhansky IS

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In this talk we will give a review of a range of logarithmic inequalities on a group. This will include log-Sobolev, log-Hardy, log-Gagliardo-Nirenberg inequalities, Nash inequalities, and their versions.

Hörmander's Theorem for Translation Invariant Operators on Banach Function Spaces

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According to [1, Theorem 1.1], there are no non-zero bounded linear translation invariant operators from $L^p(\mathbb{R}^d)$ to $L^q(\mathbb{R}^d)$ if $1 \leq q < p < \infty$. We provide conditions that ensure that are no non-zero bounded linear translation

invariant operators from a Banach function space $X(\mathbb{R}^d)$ to a Banach function space $Y(\mathbb{R}^d)$, and derive corollaries for weighted Lebesgue spaces, variable Lebesgue spaces, Lorentz-Zygmund spaces and general rearrangement-invariant spaces.

References
[1] L. Hörmander. *Estimates for translation invariant operators in L^p spaces*. Acta Math., **104**, 93–140, 1960. DOI: 10.1007/BF02547187.

Almost Everywhere Convergence of Partial Sums and Certain Summability Methods of Trigonometric and Vilenkin Systems

Lars-Erik Persson[1], ***Ferenc Schipp***[2], ***George Tephnadze***[3], ***Ferenc Weisz***[2]

IS

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The classical theory of the Fourier series deals with the decomposition of a function into sinusoidal waves. Unlike these continuous waves, the Vilenkin (Walsh) functions are rectangular waves. There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, combined with martingale theory.

This talk is devoted to investigating tools which are used to study almost everywhere convergence of the partial sums of trigonometric and Vilenkin systems. In particular, these methods combined with martingale theory help to give a simpler proof of an analogy of the famous Carleson-Hunt theorem for the Fourier series with respect to the Vilenkin system. We also define an analogy of Lebesgue points for integrable functions and we will describe which certain summability methods are convergent in these points.

References
[1] L.-E. Persson, F. Schipp, G. Tephnadze and F. Weisz, An analogy of the Carleson-Hunt theorem with respect to Vilenkin systems, *J. Fourier Anal. Appl.*, 28, 48 (2022), 1-29.
[2] L. E. Persson, G. Tephnadze and F. Weisz, *Martingale Hardy Spaces and Summability of Vilenkin-Fourier Series*, Birkhäuser/Springer, 2022, (to appear).

Asymptotics of Block Toeplitz Determinants with Piecewise Continuous Functions

Jani Virtanen[1,2]

IS

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I discuss recent results on the asymptotics of the block Toeplitz determinants generated by matrix-valued piecewise continuous functions with finitely many jumps under mild additional conditions. The approach is based on a new localization theorem for Toeplitz determinants, a new method of computing the Fredholm index of Toeplitz operators with piecewise continuous matrix-valued symbols, and other operator theoretic methods. As an application of our results, we consider piecewise continuous symbols that arise in the study of entanglement entropy in quantum spin chain models. This is joint work with Estelle Basor and Torsten Ehrhardt.

Contributed Talks (CT)

Approximation by Nörlund Means with Respect to Walsh System in Lebesgue Spaces

Nika Areshidze[1], ***George Tephnadze***[2] **CT**

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The classical theory of the Fourier series deals with the decomposition of a function into sinusoidal waves. Unlike these continuous waves, the Vilenkin (Walsh) functions are rectangular waves. The development of the theory of the Vilenkin-Fourier series has been strongly influenced by the classical theory of trigonometric series. Because of this, it is inevitable to compare the results of the Vilenkin series to those of the trigonometric series. There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, which studies orthonormal systems from the point of view of the structure of a topological group. Some important steps in early development can be found in the book by F. Schipp, W. R. Wade, P. Simon and J. Pál [3] from 1990. The research continued intensively also after this. Some of the most important steps in these developments are presented in the recent book by L. E. Persson, G. Tephnadze and F. Weisz [4] from 2022.

This talk is devoted to improving and complementing a result by Móricz and Siddiqi [2]. In particular, we prove that their estimate of the Nörlund means with respect to the Walsh system holds also without their additional condition. Moreover, we prove a similar approximation result in Lebesgue spaces for any $1 \leq p < \infty$ (for details see [1]).

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Toeplitz Operators on Large Fock Spaces

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In this talk, we give a complete characterization of the bounded and compact Toeplitz operators between different large Fock spaces F^p_ω and F^q_ω, with $0 < p, q \leq \infty$, in terms of average functions, certain generalized Berezin transforms and Carleson measures. We characterize the essential norms of Toeplitz operators from F^p_ω into F^q_ω, for $0 < p, q \leq \infty$, as well.

Maximal Operators of Partial Sums of Walsh-Fourier Series in the Martingale Hardy Spaces

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The classical theory of the Fourier series deals with the decomposition of a function into sinusoidal waves. Unlike these continuous waves, the Vilenkin (Walsh) functions are rectangular waves. The development of the theory of the Vilenkin-Fourier series has been strongly influenced by the classical theory of trigonometric series. Some important steps in early development can be found in the book by F. Schipp, W. R. Wade, P. Simon and J. Pál [4] from 1990. Some of the most important steps in the development of the theory of martingale Hardy spaces are presented in the book [5] by F. Weisz from 1994. The research continued intensively also after this. Some of the most important steps in these developments are presented in the recent book [3].

This talk is devoted to introducing some new weighted maximal operators of the partial sums of the Walsh-Fourier series with some "optimal" weights and investigating $(H_p - H_p)$ and $(H_p - weak - H_p)$ type inequalities for these new operators, for $0 < p < 1$. Moreover, we also show the sharpness of this result. As a consequence, we obtain some new and well-known results (for details see [1] and [2]).

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Summability of Tkebuchava's Means of One and Two Dimensional Trigonometric Fourier Series

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The talk is devoted to characterising the set of convergence of the general logarithmic means of the trigonometric Fourier series and also establishing a condition that guarantees convergence in the measure of logarithmic means of the two-dimensional Fourier series. We also consider the summability of two-dimensional Tkebuchava means.

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Mathematical Problems of Dynamical Interaction of Fluids and Multiferroic Solids

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We consider a three-dimensional mathematical problem of fluid-solid dynamical interaction when an anisotropic elastic body occupying a bounded region Ω^+ is immersed in an inviscid fluid occupying an unbounded domain $\Omega^- = \mathbb{R}^3 \setminus \overline{\Omega}^+$. In the solid region, we consider the generalized Green–Lindsay's model of the thermo-electro-magneto-elasticity theory. In this case, in the domain Ω^+ we have a six-dimensional thermo-electro-magneto-elastic field (the displacement vector with three components, electric potential, magnetic potential, and temperature distribution

function), while we have a scalar acoustic pressure field in the unbounded domain Ω^-. The physical kinematic and dynamical relations are described mathematically by the appropriate initial and boundary-transmission conditions. Using the Laplace transform, the dynamical interaction problem is reduced to the corresponding boundary-transmission problem for elliptic pseudo-oscillation equations containing a complex parameter τ. We derive the appropriate norm estimates with respect to the complex parameter τ and construct the solution of the original dynamical problem by the inverse Laplace transform. As a result, we prove the uniqueness, existence, and regularity theorems for the dynamical interaction problem. Actually, the present investigation is published in [1] which is a continuation of the paper [2], where the fluid-solid interaction problems for elliptic pseudo-oscillation equations associated with the above-mentioned generalized thermo-electro-magneto-elasticity theory are studied by the potential method and the theory of pseudo-differential equations.

This is joint work with D. Natroshvili.

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Solvability, Asymptotic Analysis and Regularity Results of Dynamical Interface Crack Type Transmission Problems of the Generalized Thermo-Electro-Magneto-Elasticity for Composed Structures

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We study interaction dynamical interface crack type transmission problems between thermo elastic and generalized thermo-electro-magneto elastic homogeneous anisotropic bodies. The model of generalized thermo-electro-magneto elasticity is based on Green-Lindsay's theory. This theory permits the propagation of thermal waves at finite speed. Using the Laplace transform, potential theory and the method of boundary pseudo-differential equations, we prove the existence and uniqueness of solutions. The asymptotics of solutions are obtained near the interface crack edge, and on the basis of asymptotic analysis, we establish optimal Hölder smoothness results for solutions.

This is joint work with D. Natroshvili and T. Buchukuri.

On Spectral Factorization of Rational Matrix Functions

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Spectral factorization is the process by which a positive (scalar or matrix-valued) function S is expressed in the form

$$S(t) = S_+(t)S_+^*(t), \quad t \in \mathbb{T},$$

where S_+ can be analytically extended inside the unit circle $\mathbb{T}$ and S_+^* is its Hermitian conjugate. There are multiple contexts in which this factorization naturally arises, e.g., linear prediction theory of stationary processes, optimal control, digital communications, etc. Spectral factorization is used to construct certain wavelets and multiwavelets as well. Therefore, many authors contributed to the development of different computational methods for spectral factorization. Unlike the scalar case, where an explicit formula exists for factorization, in general, there is no explicit expression for spectral factorization in the matrix case. The existing algorithms for approximate factorization are, therefore, more demanding in the matrix case.

The Janashia-Lagvilava algorithm [1], [2] is a relatively new method of matrix spectral factorization which proved to be effective [3], [4] and provides several generalizations [5], [6]. Nevertheless, the algorithm, as it was designed so far,

was not able to factorize exactly even simple polynomial matrices. In the proposed work, we cast a new light on the capabilities of the method eliminating the above-mentioned flaw. In particular, we can factorize explicitly matrices whose rational entries in the lower-upper triangular factorization can be determined (indicating their poles inside $\mathbb{T}$ and the principle parts at these poles). This extension allows the construction of rational paraunitary filter banks with preassigned poles and zeros. These filter banks are multidimensional lossless infinite impulse response filters that play an important role in linear time-invariant systems.

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On Frictional Unilateral Contact in Finite Strain Elasticity

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Using the polyconvexity approach of J. Ball [1] to nonlinear finite strain elasticity we extend the work on unilateral contact in [2,3] to include frictional phenomena along the boundary of the elastic body. We combine penalization of the unilateral constraint with regularization of the friction term [4].

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Real Analytic Bergman Spaces

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The usual theory of Bergman spaces begins with a domain D with weight ω. One takes the elements of $L^p(D)$ which are holomorphic. One could also take the real parts of such functions and study them. Two characteristic properties of Bergman spaces are that all functions are real analytic, and bounded point evaluation holds. These are well-known examples. In this talk, we consider much larger closed subspaces of functions in $L^p(C,\omega)$ where all functions are real analytic, and bounded point evaluation holds. The functions are expressed as power series in z and $\overline{z}g(z)$, where $g(z)$ is an entire function whose zeros spread out toward infinity in a certain way. This is a mild condition which is

easy to satisfy. For instance, $g(z) = \dfrac{\sin\ z \sin \tau z \sin \tau^2 z}{z^2}$ works, where $\tau^3 = 1$, $\tau \neq 1$. The proof involves CR wedge extension.

Also, work in progress towards a related theorem about functions on the unit ball in $\mathbb{C}^2$ is announced.

Banach Algebras of Convolution Type Operators with PQC Data

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Given $p \in (1, \infty)$ and a Muckenhoupt weight $w \in A_p(\mathbb{R})$, let $\mathcal{B}_{p,w}$ be the Banach algebra of all bounded linear operators acting on the weighted Lebesgue space $L^p(\mathbb{R}, w)$. Let $PQC_{\mathbb{R}}$ be the C^*-algebra all piecewise quasicontinuous functions on the real line $\mathbb{R}$ that is equivalent to piecewise slowly oscillating functions at infinity and let $PQC_{\mathbb{R},p,w}$ be the corresponding Banach algebra of Fourier multipliers on the space $L^p(\mathbb{R}, w)$. For a subclass of weights w, the Banach algebras $\mathfrak{A}_{p,w} \subset \mathcal{B}_{p,w}$ generated by all multiplication operators aI with $a \in PQC_{\mathbb{R}}$ and by all convolution operators $W^0(b)$ with $b \in PQC_{\mathbb{R},p,w}$ are studied. First, a Fredholm symbol calculus is constructed for the Banach algebra $\mathcal{Z}_{p,w} \subset \mathfrak{A}_{p,w}$ generated by the operators $aW^0(b)$ with quasicontinuous data functions $a \in QC_{\mathbb{R}}$ and $b \in QC_{\mathbb{R},p,w}$ that are equivalent to some slowly oscillating functions at infinity. Then a Fredholm symbol calculus for the Banach algebra $\mathfrak{A}_{p,w}$ is constructed and a Fredholm criterion for the operators $A \in \mathfrak{A}_{p,w}$ is established by applying the Allan-Douglas local principle, limit operators techniques and the two idempotents theorem.

The talk is partially based on joint work with C. A. Fernandes and A. Yu. Karlovich.

The Infinite Hilbert Matrix on Spaces of Analytic Functions

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The (finite) Hilbert matrix is arguably one of the single most well-known matrices in mathematics. The infinite Hilbert matrix $\mathcal{H}$ was introduced by David Hilbert around 120 years ago in connection to his double series theorem. It can be interpreted as a linear operator on spaces of analytic functions in the open unit disc of the complex plane by its action on their Taylor coefficients. The boundedness of $\mathcal{H}$ on the Hardy spaces H^p for $1 < p < \infty$ and Bergman spaces A^p for $2 < p < \infty$ was established by Diamantopoulos and Siskakis. The exact value of the operator norm of $\mathcal{H}$ acting on the Bergman spaces A^p for $4 \leq p < \infty$ was shown to be $\frac{\pi}{\sin(2\pi/p)}$ by Dostanic, Jevtic and Vukotic in 2008. The case $2 < p < 4$ was an open problem until 2018 it was shown by Bozin and Karapetrovic that the norm has the same value also on the scale $2 < p < 4$. In this talk, we review some of the old results and consider the still partly open problem regarding the value of the norm on weighted Bergman spaces. We also consider a generalised Hilbert matrix operator and its (essential) norm. The talk is partly based on a joint work with Mikael Lindström, David Norrbo and Niklas Wikman.

On the Convergence of Cesàro Means of Negative Order of Vilenkin-Fourier Series

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In the paper by Onneweer and Waterman [1], they established a sufficient condition that guarantees the uniform convergence of the Vilenkin-Fourier series for continuous functions. The focus of my talk centres around the investigation of distinct function classes characterized by properties associated with generalized bounded oscillation. Within the scope of these function classes, we derive conditions that ensure the uniform convergence of Cesàro means with negative order [2].

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Comparative Analysis of Approximate Solutions for a Numerical Solution of a Two-Dimensional Nonlinear Model

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An approximate solution of the initial-boundary value problem for one system of non-linear two-dimensional partial differential equations [1] is considered. Such type models are studied in many works (see, for example, [2-5] and references therein). We constructed a variable direction difference scheme and σ-weighted difference scheme of the averaged type. We implemented the relevant algorithms and presented the data obtained as a result of numerical experiments and relevant comparative analysis.

Acknowledgement
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Properties of the Sequence of Linear Functionals on BV with Application

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The talk is devoted to investigating the sequence of some linear functions in the space BV of finite variation functions. We prove that under certain conditions this sequence is bounded. We also prove that this result is sharp. In particular, the obtained results can be used to study the convergence of some general Fourier series. Moreover, the obtained conditions seem to be new and useful also for classical orthonormal systems.

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On a Convergence of a Three-Layer Semi-Discrete Approach for Solving the Nonlinear Dynamic Kirchhoff-Type String Equation with Time-Varying Coefficients

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In this talk, we explore an initial-boundary value problem (IBVP) linked to the nonlinear dynamic string equation of Kirchhoff-type. This equation includes time-dependent coefficients and has been discussed in the work [1]. Our aim is to develop a method for discretizing time that can effectively estimate the solution to the IBVP.

To achieve this goal, we employ a symmetrical three-layer semi-discrete technique concerning the time variable. In this approach, we evaluate the nonlinear term at the middle node points, simplifying the computation of numerical solutions at each time step by inverting linear operations. As a result, this leads to a system of second-order linear ordinary differential equations.

We have demonstrated the local convergence of the proposed strategy, which reveals a quadratic rate of convergence in relation to the time discretization step within the local time interval. Lastly, we have performed several numerical experiments using the suggested method for various test scenarios. The obtained numerical results align well with the theoretical findings.

Acknowledgement
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Useful Information

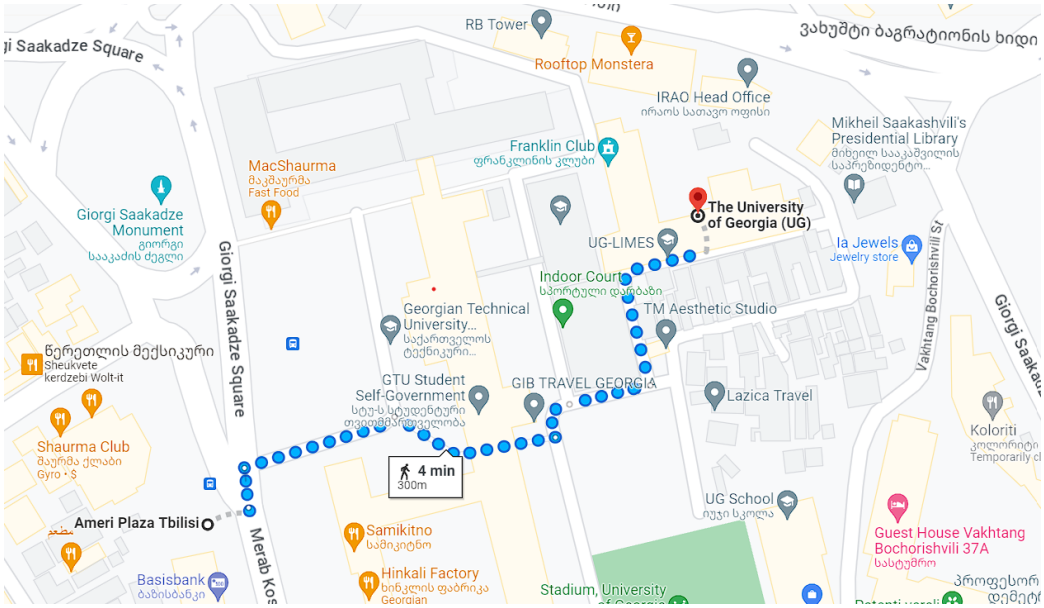
If you require a **taxi**, please be aware that **taxis** in Tbilisi are typically unmetered, and as a result, tourists can occasionally fall victim to overcharging. This makes hailing taxis from the street a somewhat risky endeavour. We highly recommend downloading a ride-hailing app such as **Bolt** and using it to book your taxis in Tbilisi. A journey between the airport and the city centre, or vice versa, when booked through **Bolt**, should commonly not exceed *10 Euros* in cost.

For detailed and useful information about public transportation in Tbilisi, you can visit the following link: https://ttc.com.ge/en.

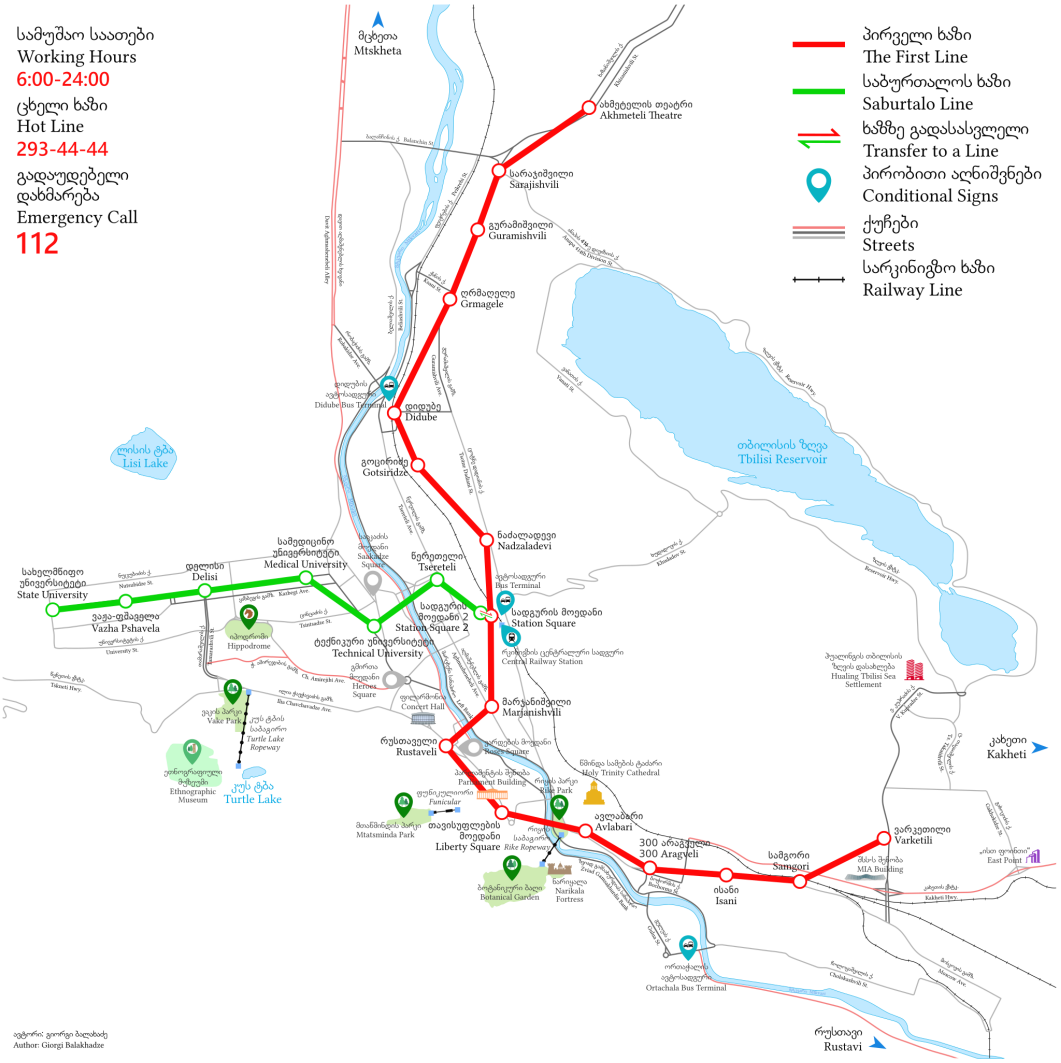
For lunch, we recommend the restaurants **"Samikitno-Matchakhela"** and **"Khinkali Factory"**, which are located on the opposite side of the **Ameri Plaza Hotel Tbilisi** in close proximity to the **University of Georgia (UG)**. You can find them at the following address: 77 Merab Kostava St., Tbilisi 0171. If you prefer a food delivery service to your location, Tbilisi offers several international food delivery apps, such as **Glovo**, **Wolt**, and **Bolt Food**.

Like many countries, most restaurants, supermarkets, and other establishments in our region accept **credit** or **debit card** payments (**Amex**, **Visa**, or **MasterCard**). However, if you prefer to use **cash**, we recommend exchanging currency on **Pekini Ave.**, which is conveniently located near the **conference venue**. Please be aware that exchanging currency at **banks**, **airports** and in **touristic zones** (such as the Liberty Square neighbourhood) might not provide favourable exchange rates for you.

The route from the **Ameri Plaza Hotel Tbilisi** to the conference venue, **the University of Georgia (Tbilisi)**, takes approximately **4-5 minutes** on foot and covers a distance of about **300 meters**. You can view the route on Google Maps by following this link: https://goo.gl/maps/bZswfPAa7oxJTfi99.



The closest underground station to the conference venue is the **Technical University** station. See the map below



In the event of an **emergency**, please call **112**.

Partner Institutions and Sponsors

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