TAPDE 2024

Program & Abstracts

The Tbilisi Analysis & PDE Workshop

August 28 - August 31, 2024

The University of Georgia, Tbilisi

The book of abstracts has been assembled by *Zurab Vashakidze*
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The digital edition of this booklet is accessible through the following link:
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The organizers of the **TAPDE workshop** wish to express their sincere gratitude to **Arne Hendrickx** for his assistance in designing the cover for this book of abstracts and the poster for the workshop, with the aid of the *Artificial Intelligence (AI)* tool *Microsoft Copilot*.

The most recent update of the **Program** and the **Book of Abstracts** was completed on **August 25, 2024**.

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About

This is a generic version of the real AMCOS conference booklet for which this LaTeX template was generated. All information about the use and distribution of this template, and all related codes, can be found at `https://github.com/maximelucas/AMCOS_booklet`.

TAPDE2024

The Tbilisi Analysis & PDE Seminar (TAPDE) was established in November 2020 as a bi-weekly series of online seminars, featuring presentations from over 50 distinguished scholars from various countries. Although the initial plan included complementing the online talks with an annual workshop, this was postponed due to the COVID-19 pandemic until 2023, when the first **Tbilisi Analysis & PDE Workshop** (TAPDE2023)[1] was held in Tbilisi, the capital of Georgia. This workshop has now become an annual international event.

The workshop covers a broad range of topics, including Real and Complex analysis, Operator Theory, Harmonic Analysis, Integral Equations, Numerical Analysis, Partial Differential Equations, Mathematical Physics, and related fields. We invite researchers to submit their work for presentation, share their latest research findings, and engage in discussions about new developments in their respective areas of expertise.

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[1] https://tapde-workshop.ug.edu.ge/2023/

Timetable

Invited Talks (IT): The invited talk is scheduled to last **60 minutes**, with **5 minutes** included for questions.
Contributed Talks (CT): The contributed talk is scheduled for **30 minutes**, including **5 minutes** for questions.

August 28, Wednesday, Session at the University of Georgia, Tbilisi

08:30–09:00	**Registration of Participants, Room 519**		
09:00–09:30	**Opening Ceremony, Room 519**		
Room 519	**Plenary Session**		
Chairman	**Guillermo P. Curbera**		
09:30–10:20	IT	**Alex Iosevich** Rochester, New York, United States	Uncertainty Principles, Signal Recovery and Restriction Theory
10:30–11:20	IT	**Elijah Liflyand** Ramat-Gan, Israel	Variation Type Characterization of Product Hardy Spaces
11:30–12:00	**Coffee Break**		
Room 222	**Semi-Plenary Session**		
Chairman	**Ilya M. Spitkovsky**		
12:00–12:50	IT	**Gennady Mishuris** Aberystwyth, Wales, United Kingdom	On Wiener-Hopf Factorization of Matrix Functions: 'Regularised' Vs. 'Exact' Factorization
Room 201	**Semi-Plenary Session**		
Chairman	**Michael Ruzhansky**		
12:00–12:50	IT	**Giorgi Oniani** Kutaisi, Georgia	Convergence and Divergence of Fourier Series in Systems of Characters for Compact Groups
13:00–14:30	**Lunch Break**		
Online	**Plenary Session**		
Chairman	**Jean Lagacé**		
14:30–15:20	IT	**Peter Kuchment** College Station, Texas, United States	Nodal Count Mystery
Room 222	**Section of Differential Equations and Applications**		
Chairman	**Tengiz Buchukuri**		
15:30–15:55	CT	**Ashot Gevorkyan** Yerevan, Armenia	Three-Body Problem in Conformal-Euclidean Space: Hidden Symmetries and New Properties of a Low-Dimensional System
16:00–16:25	CT	**Daniel Seibel** Saarbrücken, Saarland, Germany	Analytical Integration in Integral Equation Methods
16:30–17:00	**Coffee Break**		

17:00–17:25	CT	**Zurab Vashakidze** Tbilisi, Georgia	Numerical Treatment for the Nonlinear Dynamic String Equation of Kirchhoff-Type with Time-Dependent Coefficients
17:30–17:55	CT	**Asselya Smadiyeva** Almaty, Kazakhstan	Time-Fractional Diffusion Equation
Room 201	**Section of Analysis and Applications**		
Chairman	**Alex Iosevich**		
15:30–15:55	CT	**George Tephnadze** Tbilisi, Georgia	Approximation by Matrix Transform Means with Respect to the Vilenkin System in Lebesgue Spaces
16:00–16:25	CT	**Alina Shalukhina** Lisbon, Portugal	Self-Improving Property of the Hardy-Littlewood Maximal Operator Over Spaces of Homogeneous Type
16:30–17:00	**Coffee Break**		
17:00–17:25	CT	**Karina Navarro Gonzalez** Yerevan, Armenia	Reproducing Kernel Hilbert Spaces and Covering Numbers on Compact Lie Groups
18:15	**Welcome Reception – Inauguration of the Institute of Mathematics**		

August 29, Thursday, Session at Sevsamora, Saguramo

08:00	**Departure from the front door of the University of Georgia, Tbilisi**		
Conference Room	**Plenary Session**		
Chairman	**Roland Duduchava**		
09:00–09:50	IT	**Michael Ruzhansky** Ghent, Belgium	Pseudo-Differential Operators in Groups
10:00–10:50	IT	**Marianna Chatzakou** Ghent, Belgium	Functional Inequalities on Lie Groups
11:00–11:30	**Coffee Break**		
Chairman	**Oleksiy Karlovych**		
11:30–12:20	IT	**Ilya M. Spitkovsky** Abu Dhabi, United Arab Emirates	Fifty Years of Fun with Numerical Ranges
12:30–13:20	IT	**Jani Virtanen** Finland & United Kingdom	On the Asymptotics of Determinants for Structured Matrices
13:30–14:30	**Lunch Break**		
Conference Room	**Plenary Session**		
Chairman	**Jani Virtanen**		
14:30–15:20	IT	**Oleksiy Karlovych** Lisbon, Portugal	A Necessary Condition for the Boundedness of the Maximal Operator on $L^{p}(\cdot)$ Over Reverse Doubling Spaces of Homogeneous Type
15:30–16:20	IT	**Guillermo P. Curbera** Seville, Andalucia, Spain	The Finite Hilbert Transform Taking Values in the Zygmund Space L_{exp}
16:30–19:00	**Excursion to Mtskheta (Optional)**		
19:00	**Conference Dinner at “Kartuli Sakhli”**		

The restaurant **"Kartuli Sakhli"** (in English, **Georgian House**) is located at **2 Giorgi Tsabadze Street, Tbilisi 0112, Georgia**. You can find it on Google Maps via the following link: https://maps.app.goo.gl/uDLQ8GgUszMmMJkj8. Please note that dinner at the restaurant is optional and costs **20 Euros** per person, with a portion of the cost *subsidized* by the **conference budget**. Additionally, there is no ***formal dress code*** for the restaurant.

August 30, Friday, Day off, Excursion

09:00–19:00	**Excursion to Kakheti, Departure from the Front Door of the University of Georgia, Tbilisi**

The **Kakheti excursion** is an optional event with a fee of **15 Euros** per person. This fee, like that of the **conference dinner**, is partially offset by a subsidy from the **conference budget**.

August 31, Saturday, Session at the University of Georgia, Tbilisi

Room 222	**Plenary Session**		
Chairman	**Elijah Liflyand**		
09:00–09:50	IT	**Jean Lagacé** London, United Kingdom	Boundary Regularity of Conformal Maps and Spectral Geometry
Room 222	**Semi-Plenary Session**		
Chairman	**Gennady Mishuris**		
10:00–10:50	IT	**Arsen Pskhu** Nalchik, Kabardino-Balkaria, Russian Federation	On the Theory of Distributed Order Differentiation Operators
Room 201	**Semi-Plenary Session**		
Chairman	**Zurab Vashakidze**		
10:00–10:50	IT	**Temur Jangveladze** Tbilisi, Georgia	Some Nonlinear Partial Differential and Parabolic Type Integro-Differential Models Based on System of Maxwell Equations
11:00–11:30	**Coffee Break**		
Online	**Plenary Session**		
Chairman	**Marianna Chatzakou**		
11:30–12:20	IT	**Duván Cardona** Ghent, Belgium	Control Theory for the Heat Equation for Non-Local Elliptic Pseudo-Differential Operators on Compact Lie Groups
12:30–14:00	**Lunch Break**		
Room 222	**Section of Differential Equations and Applications**		
Chairman	**Eugene Shargorodsky**		
14:00–14:25	CT	**Tengiz Buchukuri** Tbilisi, Georgia	Dynamical Boundary-Transmission Problems of the Generalized Thermo-Electro-Magneto-Elasticity Theory for Composed Structures with Interface and Interior Cracks
14:30–14:55	CT	**Alexander Oleinikov** Tbilisi, Georgia	Strength Theory of Layered Fibrous Polymer Composites

15:00–15:25	CT	**Ainur Zholamankyzy** Almaty, Kazakhstan	On the Existence of the Solution to the Goursat Problem for a Loaded System of Hyperbolic Equations
15:30–15:55	CT	**Medea Tsaava** Tbilisi, Georgia	Transmission Problems for a Second Order Differential Equation on a Hypersurface with Lipschitz Boundary in the Generic Bessel Potential Spaces
Room 201	**Section of Analysis and Applications**		
Chairman	**George Tephnadze**		
14:00–14:25	CT	**Monire Mikaeili Nia** Yerevan, Armenia	Strong and Weak Type Estimates for the Littlewood-Paley Operator g^*_λ, ψ with Non-Convolution Kernel
14:30–14:55	CT	**Zahra Keyshams** Yerevan, Armenia	On the Schur-Horn Problem
15:00–15:25	CT	**Davit Baramidze** Georgia	Some New Restricted Maximal Operators of Fejér Means of Walsh-Fourier Series
15:30–15:55	CT	**Sergey Tikhonov** Barcelona, Spain	Marcinkiewicz–Zygmund Inequalities in (Quasi-)Banach Spaces
16:00–16:10	**Closing**		
16:10	**Farewell**		

List of Abstracts – Talks

Invited Talks (IT)

Control Theory for the Heat Equation for Non-Local Elliptic Pseudo-Differential Operators on Compact Lie Groups

Duván Cardona[1]

IT

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In this talk we discuss our recent results about spectral inequalities for eigenvalues and their applications to control theory. In [1] we extend the estimates proved by Donnelly and Fefferman, and by Lebeau and Robbiano for sums of eigenfunctions of the Laplacian (on a compact manifold) to estimates for sums of eigenfunctions of any positive and elliptic pseudo-differential operator of positive order on a compact Lie group. Our criteria are imposed in terms of the positivity of the corresponding matrix-valued symbol of the operator. As an application of these inequalities, we obtain the null-controllability for diffusion models for elliptic pseudo-differential operators on compact Lie groups. General results are also discussed on compact manifolds. Joint work with Michael Ruzhansky and Julio Delgado.

References

[1] D. Cardona, J. Delgado, M. Ruzhansky, Estimates for sums of eigenfunctions of elliptic pseudo-differential operators on compact Lie groups, to appear in *J. Geom. Anal.*
[2] H. Donnelly, C. Fefferman, Nodal domains and growth of harmonic functions on noncompact manifolds. *J. Geom. Anal.* **2(1)**, (1992), 79–93.
[3] H. Donnelly, C. Fefferman, Nodal sets of eigenfunctions on Riemannian manifolds, *Invent. Math.* **93**, (1988), 161–183.

Functional Inequalities on Lie Groups

Marianna Chatzakou[1]

IT

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We will discuss several logarithmic Sobolev-type inequalities in the setting of stratified (Carnot) Lie groups. In particular, we will see how the classical Gaussian logarithmic Sobolev inequality extends to this hypoelliptic setting. We will also present a unified approach to how to obtain the logarithmic Hardy inequality and the generalised Poincaré inequality with weights in this setting. The obtained inequalities are new even when considered in the trivial case of case of a stratified group, i.e. in the Euclidean case.

The Finite Hilbert Transform Taking Values in the Zygmund Space L_{exp}

Guillermo P. Curbera[1], ***Susumu Okada***[2], ***Werner J. Ricker***[3]

IT

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[2] Kambah, Australian Capital Territory, Australia
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We study the action of the finite Hilbert transform defined on $L^\infty(-1,1)$ and take its values in the Zygmund space $L_{\mathrm{exp}}(-1,1)$. This is a reciprocal situation to the investigation that we recently undertook of the finite Hilbert transform defined on the Zygmund space $L\log L(-1,1)$ and taking its values in $L^1(-1,1)$. The fact that both $L^\infty(-1,1)$ and $L_{\mathrm{exp}}(-1,1)$ fail to be separable generates new features not present before.

References

[1] G. P. Curbera, S. Okada, W. J. Ricker. *The finite Hilbert transform acting on the Zygmund space $L\log L$.* Ann. Sc. Norm. Super. Pisa Cl. Sci., to appear.
[2] G. P. Curbera, S. Okada, W. J. Ricker. *The finite Hilbert transform taking values in the Zygmund space L_{exp}.* preprint.

Uncertainty Principles, Signal Recovery and Restriction Theory

Alex Iosevich[1]

IT

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Let $f:\mathbb{Z}_N^d \to \mathbb{C}$ be a signal. Suppose that this signal is transmitted via its Fourier transform

$$\widehat{f}(m) = N^{-d} \sum_{x \in \mathbb{Z}_N^d} \chi(-x \cdot m) f(x).$$

Suppose that the frequencies $\{\widehat{f}(m)\}_{m \in S}$ are lost due to noise, interference or other causes. Under what circumstances can we still recover the original signal f exactly? We are going to discuss a variety of results centred around the idea that a good recovery condition is always possible if the set of missing frequencies satisfies a non-trivial Fourier restriction estimate. We bring Bourgain's celebrated Λ_p theorem to bear on this problem. We are also going to discuss improved recovery conditions using multiple transmissions. Multi-linear restriction theory plays an important role there.

Some Nonlinear Partial Differential and Parabolic Type Integro-Differential Models Based on System of Maxwell Equations

Temur Jangveladze[1]

IT

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The investigation and numerical solution of the initial-boundary value problems for some nonlinear partial differential, and parabolic type integro-differential models are considered. The models are based on the well-known system of Maxwell equations which describes the process of propagation of an electromagnetic field into a medium [1].

The existence, uniqueness and asymptotic behaviour of solutions, as time tends to infinity, for some types of initial-boundary value problems are studied. The examples of one-dimensional nonlinear systems and their analytical solutions are given which show that those systems, in general, do not have global solutions. Consequently, the case of a blow-up solution is observed. Linear stability of the stationary solution of the initial-boundary value problem for one nonlinear system is proved. The possibility of occurrence of the Hopf-type bifurcation is established. Semi-discrete and finite difference approximations are discussed [2].

Complex nonlinearity dictates also splitting the investigated equations along the physical process and studying the basic model by split ones, where the first model considers the Joule law, whereas the second process deals with heat conductivity [3]. The splitting-up scheme with respect to physical processes for one-dimensional cases as well as additive Rothe-type semi-discrete schemes for multi-dimensional cases are investigated. The stability and convergence properties of those schemes are studied. Algorithms for finding approximate solutions are constructed.

Note that and above-mentioned integro-differential models have arisen in [4] and studied for the first time in [4], [5]. Based on the works [4], [5], one integro-differential model appeared in G. Laptev's investigation, and the author named those models as averaged integro-differential equations. Publications [4], and [5] quickly attracted the attention of scientists. For relatively complete citations on this issue up to 2019, see the monographs [2], [6]. Since then, this interest has not been reduced but increased.

Besides the essential nonlinearity, the complexities of the above-mentioned models are caused by their multi-dimen-sionality. It is well known that the general method for constructing economic algorithms for multi-dimensional problems of mathematical physics is the method of decomposition. This approach allows to reduce of multi-dimensional problems to a set of one-dimensional ones, whose numerical realizations obviously need less computer resources (see, for example, [2] and the references therein).

The purpose of the present talk is to continue our study and give a description of both the results obtained from the research and the numerical resolution of nonlinear partial differential and integro-differential models based on the systems of Maxwell equations.

Acknowledgement
This research has been supported by the Shota Rustaveli National Science Foundation of Georgia under the grant FR-21-2101.

References

[1] L. D. Landau and E. M. Lifshic, Electrodynamics of Continuous Media. (Russian) Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1957.
[2] T. Jangveladze. Investigation and Numerical Solution of Nonlinear Partial Differential and Integro-Differential Models Based on System of Maxwell Equations. Mem. Differential Equations Math. Phys., 76, 1-118, 2019.
[3] I. O. Abuladze, D. G. Gordeziani, T. A. Jangveladze and T. K. Korshiya, Discrete models for a nonlinear magnetic-field scattering problem with thermal conductivity. (Russian) Differ. Uravn., 22, 7, 1119–1129, 1986; translation in Differ. Equ., 22, 7, 769–777, 1986.
[4] D. G. Gordeziani, T. A. Jangveladze and T. K. Korshiya, Existence and uniqueness of the solution of a class of nonlinear parabolic problems. (Russian) Differ. Uravn., 19, 7, 1197–1207, 1983; translation in Differ. Equ., 19, 887-895, 1983.
[5] T. A. Jangveladze, The first boundary value problem for a nonlinear equation of parabolic type. (Russian) Dokl. Akad. Nauk SSSR, 269, 4, 839-842, 1983; translation in Sov. Phys., Dokl., 28, 323-324, 1983.
[6] T. Jangveladze, Z. Kiguradze and B. Neta. Numerical Solutions of Three Classes of Nonlinear Parabolic Integro-Differential Equations. Elsevier/Academic Press, Amsterdam, 2016.

A Necessary Condition for the Boundedness of the Maximal Operator on $L^p(\cdot)$ Over Reverse Doubling Spaces of Homogeneous Type

IT

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Let (X, d, μ) be a space of homogeneous type and $p(\cdot) : X \to [1, \infty]$ be a variable exponent. We show that if the measure μ is Borel-semiregular and reverse doubling, then the condition $\operatorname*{ess\,inf}_{x \in X} p(x) > 1$ is necessary for the boundedness of the Hardy-Littlewood maximal operator M on the variable Lebesgue space $L^{p(\cdot)}(X, d, \mu)$. The talk is based on the paper [1].

References

[1] O. Karlovych, A. Shalukhina, *A necessary condition for the boundedness of the maximal operator on $L^{p(\cdot)}$ over reverse doubling spaces of homogeneous type*, Analysis Mathematica, accepted, URL: `https://arxiv.org/abs/2403.10915`.

Nodal Count Mystery

IT

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The classical Sturm's theorem claiming that in 1D the n-th eigenfunction of the Sturm-Liouville operator has exactly n nodal domains (i.e. those where it preserves the sign) is equal to n. Its "natural" multi-dimensional version, as it has been known for a century, fails manifestly. Only in the last one and a half decades did an understanding of this nodal count emerge. The talk will present a survey of this recent progress.

Boundary Regularity of Conformal Maps and Spectral Geometry

IT

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In the spectral geometry of surfaces, it is often wise to transpose the spectral problem, for instance, the Neumann or the Steklov problem, on an arbitrary surface with a boundary to a weighted spectral problem on a circle domain in a surface of constant curvature via a conformal change of variable. This procedure transfers considerations of the regularity of the surface and its boundary to the regularity of the weight arising from the conformal factor. As such, we need to know, depending on the original surface, what is the regularity and integrability class of this conformal factor globally, i.e. up to the boundary. Classical results in Geometric Function Theory study the dependence of the conformal map on the regularity of the boundary, but they are concerned with measures of continuity that are not suitable for harmonic analysis techniques that are used in obtaining spectral estimates.

I will present work in progress where we generalise the celebrated Kellogg–Warschawski theorem, stating that if the boundary of a simply connected domain Ω has regularity $C^{k,\alpha}$, then any conformal map Φ :

$\mathbb{D} \to \Omega$ guaranteed by the Riemann mapping theorem extends to a bi-$C^{k,\alpha}$ homeomorphism $\overline{\mathbb{D}} \to \overline{\Omega}$. We generalise this theorem to the Sobolev scale, where we show that for any $s > 2$, if the boundary of Ω is the image of the circle by a $W^{s+1,p}$ map of an annulus $\{1-\varepsilon < |z| < 1+\varepsilon\}$, then any conformal map $\Phi : \Omega \to \mathbb{D}$ extends to a bi-$W^{s+1,p}$ homeomorphism $\overline{\mathbb{D}} \to \overline{\Omega}$. I will then discuss how this can be further used to obtain precise regularity-dependent asymptotics for the Steklov problem.

This is based on various joint works with Alix Deleporte (Paris-Saclay), Mikhail Karpukhin (UCL), Leonid Parnovski (UCL), and Iosif Polterovich (Montréal).

Variation Type Characterization of Product Hardy Spaces

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IT

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In the multidimensional Euclidean space, besides the classical real Hardy space $H^1(\mathbb{R}^n)$, there are numerous product Hardy spaces defined as

$$H^1_m(\mathbb{R}^n) := H^1(\mathbb{R}^{n_1} \times \cdots \times \mathbb{R}^{n_m}),$$

where $n_1 + n_2 + \cdots + n_m = n$. The number of such spaces is given by

$$\sum_{m=2}^{n} \frac{1}{m!} \sum_{k=0}^{m} (-1)^{m-k} \binom{m}{k} k^n = \sum_{k=1}^{n} (-1)^k k^n \sum_{m=k}^{n} \frac{(-1)^m}{m!} \binom{m}{k} - 1.$$

Trivially, $H^1_1(\mathbb{R}^n) = H^1(\mathbb{R}^n)$. The remaining spaces are of significant interest in Fourier Analysis. They can be defined via Riesz transforms and their combinations associated with the corresponding groups of variables. For the case where $m = n$ and $n_1 = \cdots = n_n = 1$, we denote this as

$$H^1(\mathbb{R} \times \cdots \times \mathbb{R}) := H^1_n(\mathbb{R} \times \cdots \times \mathbb{R}).$$

In this scenario, Riesz transforms become Hilbert transforms, and combinations of Hilbert transforms replace those of Riesz transforms.

We associate with each of these spaces a class of functions related to known and new variations. This characterization is achieved through the integrability of the Fourier transform. While $H^1(\mathbb{R} \times \cdots \times \mathbb{R})$ corresponds to Hardy's variation, $H^1(\mathbb{R}^n)$ is associated with Tonelli's variation. For the intermediate product Hardy spaces between $H^1(\mathbb{R}^n)$ and $H^1(\mathbb{R} \times \cdots \times \mathbb{R})$, we introduce spaces characterized by properties resembling variation. These two scales are equal in number, with a unique correspondence between their elements. Our main result states that:

If f belongs to a space defined by certain conditions involving absolute continuity of lower derivatives, and the corresponding n-th derivative belongs to the related product Hardy space, then the Fourier transform of f is Lebesgue integrable.

Notably, the derivatives are considered with respect to the same variables in which the product Hardy space is defined. Given the well-known relation between absolute continuity and variation, these spaces are naturally labelled as "variation-type spaces".

This work is a joint effort with Laura Angeloni and Gianluca Vinti.

On Wiener-Hopf Factorization of Matrix Functions: ‘Regularised’ Vs. ‘Exact’ Factorization

***Lasha Ephremidze*[1,2], *Gennady Mishuris*[3], *Ilya M. Spitkovsky*[1]**

IT

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For an $r \times r$ matrix function $S(t)$ defined on the unit circle $\mathbb{T}$, its (right) Wiener-Hopf factorization is the representation:

$$S(t) = S^{+}(t)\Lambda(t)S_{-}(t), \tag{1}$$

where $S^{\pm}$, along with their inverses, can be extended analytically in $\mathbb{T}_{+} = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathbb{T}_{-} = \{z \in \mathbb{C} : |z| > 1\} \cup \{\infty\}$, respectively, and $\Lambda(t) = \mathrm{diag}(t_1^{\varkappa}, t_2^{\varkappa}, \ldots, t_r^{\varkappa})$, $\varkappa_j \in \mathbb{Z}$, $j = 1, 2, \ldots, r$, is a diagonal matrix. The integers $\varkappa_j$ are called (right) partial indices of S. We denote the ordered set of partial indices of S ($\varkappa_1 \geq \varkappa_2 \geq \ldots \geq \varkappa_r$) by $\mathcal{PI}(S)$. The set $\mathcal{PI}(S)$ is uniquely defined. However, if $\max \mathcal{PI}(S) > \min \mathcal{PI}(S) + 1$, the factorization (1) is unstable, i.e., small perturbation of the matrix S can change the set $\mathcal{PI}(S)$ [1].

This creates a major obstacle to the application of the factorisation technique in practice. Only recently, some results on *e*xact factorisation of matrix polynomials have been delivered eliminating this obstacle [2,3]. Moreover, an effective criterion for a stable set of partial indices has been proposed [4].

On the other hand, if the matrix comes from an application, measurement error or an incomplete process description may force one to deal with an approximation $\hat{S}$ instead of the original matrix function S. Then, searching for an *e*xact factorization of matrix $\hat{S}$ may not make sense, especially considering that the latter process is rather computationally demanding. Here, the following observation might be insightful [5]: If $S_n \to S$ and $\mathcal{PI}(S_n) \neq \mathcal{PI}(S)$, then $\|S_n^{+}\|$ or $\|S_n^{-}\|$ blows up when $n \to \infty$. This calls for a concept of the *r*egularized factorization of matrix functions.

Although the set $\mathcal{PI}(S)$ is usually unknown, assuming the factors $S^{\pm}$ in the factorisation of the original matrix function S are ‘reasonably’ bounded, we can search for an *e*xact factorisation of the approximate matrix functions S_n: $\|S_n - S\| < \varepsilon$, minimising their factors $S_n^{\pm}$ with respect to various $\mathcal{PI}(S_n)$.

$$\min_{S_n \in \mathcal{S}, \|S_n - S\| < \varepsilon} \max\{\|S_n^{+}\| \|S_n^{-}\|\} = \max\{\|S_*^{+}\| \|S_*^{-}\|\}. \tag{2}$$

The approximate matrix functions S_n should belong to a suitable class $\mathcal{S}$ allowing for *e*xact factorisation. It is clear that the approximate factorisation should be properly normalised [6].

We call the $\mathcal{PI}(S_*) = \mathcal{PI}(S_*)(\mathcal{S}, \varepsilon)$ regularised partial indices of S, and $S_* = S_*^{+}\Lambda_* S_*^{-}$ the respective *r*egularized factorization of S. During the talk, we will discuss numerical methods allowing for *e*xact and *r*egularised Wiener-Hopf factorization.

Acknowledgement
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Convergence and Divergence of Fourier Series in Systems of Characters for Compact Groups

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The following topics will be discussed: Antonov-Sjölin-Soria type extrapolation in the context of locally compact groups and its application to almost everywhere convergence of Vilenkin-Fourier series; Konyagin's problem regarding the almost everywhere convergence of subsequences of Walsh-Fourier sums; Kanzelson's type theorems concerning sets of divergence of Fourier series in systems of characters for compact Abelian groups; and a multi-parameter version of Stein's weak type maximal principle, along with its applications to multiple Fourier series and ergodic averages.

The presentation is based on the author's papers [1-4].

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On the Theory of Distributed Order Differentiation Operators

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Consider the operator

$$D_{0x}^{[\mu]} f(x) = \int_{\mathbb{R}} D_{0x}^{t} f(x) \, \mu(dt) \,, \tag{1}$$

associated with a signed Borel measure μ on $\mathbb{R}$. Here D_{0x}^{t} is the Riemann-Liouville fractional derivative (or integral) of order t with respect to x with origin at $x = 0$.

Operators containing integration over the order of differentiation constitute the class of integro-differential operators of distributed order [1].

The report discusses an approach to the inversion of the operator (1). The approach is based on the generalized Stankovic transforms [2]. In particular, a Sonin pair [3,4] for the kernel of the operator (1) is found, the inverse operator is constructed, and the Newton-Leibniz formulas are proved.

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Pseudo-Differential Operators in Groups

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In this lecture we will give an overview of the global theory of pseudo-differential operators on Lie groups of different types, and the related functional and harmonic analysis.

Fifty Years of Fun with Numerical Ranges

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The numerical range $W(A)$ of an n-by-n matrix A is defined as the image of the unit sphere in $\mathbb{C}^n$ under the mapping $x \mapsto x^*Ax$. It is a compact and convex subset of $\mathbb{C}$ (the latter due to the celebrated Toeplitz-Hausdorff theorem) containing the spectrum $\sigma(A)$ of A. For normal A, $W(A)$ is the convex hull of $\sigma(A)$, while for $n = 2$, $W(A)$ is an elliptical disk with the foci at the eigenvalues of A (the Elliptical Range theorem).

Beyond these classical statements, knowledge is limited. I will discuss further results, particularly the relationship between $W(A)$ and the Kippenhahn polynomial and Kippenhahn curve $C(A)$ of A. For the class of reciprocal matrices, criteria for $C(A)$ to contain (or consist only of) elliptical components will be established. These results provide insight into the phenomenon of the existence of unitarily irreducible matrices A with elliptical $W(A)$ for arbitrary n.

Connections with the solvability theory of Toeplitz operators will be explained, and some open problems will be stated.

On the Asymptotics of Determinants for Structured Matrices

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I discuss the determinant asymptotics for structured matrices with a focus on Toeplitz and Toeplitz plus Hankel matrices, and also finite sections of Toeplitz operators, all with smooth matrix-valued symbols. Most of the results can be proved easily using operator theory but we also compare the operator-theoretic approach with the use of Riemann-Hilbert problems. Some applications to random matrix theory are also mentioned. This is joint work with Estelle Basor and Torsten Ehrhardt.

Contributed Talks (CT)

Some New Restricted Maximal Operators of Fejér Means of Walsh-Fourier Series

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The theory of the Fourier series with respect to the Walsh system deals with the decomposition of a function into rectangular waves (for details see the book [2]).

This talk is devoted to characterizing the maximal subspace of natural numbers such that the restricted maximal operator of Walsh-Fejér means on this subspace is bounded from the martingale Hardy space $H_{1/2}$ to the Lebesgue space $L_{1/2}$ (for details see the book [1]).

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Dynamical Boundary-Transmission Problems of the Generalized Thermo-Electro-Magneto-Elasticity Theory for Composed Structures with Interface and Interior Cracks

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In the talk we consider the three-dimensional dynamical mixed boundary-transmission problems for a composed body consisting of two adjacent anisotropic elastic components having a common interface surface. The two contacting elastic components are subject to different mathematical models: Green-Lindsay's model of generalized thermo-electro-magneto-elasticity in one component and Green-Lindsay's model of generalized thermo-elasticity in the other one. The composed elastic structure under consideration contains interfacial and interior cracks. We prove the uniqueness and existence theorems in appropriate function spaces and obtained smoothness and asymptotics of solutions in the neighbourhood of crack edges.

Acknowledgement

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Three-Body Problem in Conformal-Euclidean Space: Hidden Symmetries and New Properties of a Low-Dimensional System

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Despite the huge number of research into the three-body problem in physics and mathematics, the study of this problem still remains relevant both from the point of view of its broad application and taking into account its fundamental significance for the theory of dynamical systems. In addition, to solve the problem of quantum-to-classical transition, it is important to answer the question: is irreversibility fundamental to the description of the classical world? To answer this question, we considered a reference classical dynamical system, the general three-body problem, formulating it in conformal Euclidean space and rigorously proving its equivalence to the Newtonian three-body problem. It is shown that a curved configuration space with a local coordinate system reveals new hidden symmetries of the internal motion of a dynamical system, which makes it possible to reduce the problem to a 6th-order system instead of the known 8th-order [1]. The most important consequence of this consideration is that the chronologizing parameter of the motion of a system of particles, which we call internal time, is in the general case irreversible, which is characteristic of the general three-body problem. An equation is derived that describes the evolution of the flow of geodesic trajectories, with the help of which the entropy of the system is constructed. New criteria for assessing the complexity of a low-dimensional dynamic system and the dimension of stochastic fractal structures arising in three-dimensional space are obtained. An effective mathematical algorithm has been developed for the numerical simulation of the general three-body problem, which is traditionally a difficult-to-solve system of stiff ordinary differential equations.

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Reproducing Kernel Hilbert Spaces and Covering Numbers on Compact Lie Groups

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We characterize the symmetry and positivity of an integral kernel on a compact Lie group G in terms of its symbol. Next, we present the Reproducing Kernel Hilbert Space (RKHS) that the previous kernel generates, and finally, we present estimates for the covering numbers of the unit ball of RKHS of functions on a compact Lie group. The bounds we obtain depend on the dimension of the group and the rate of decay or growth of the kernel coefficients [1-3].

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On the Schur-Horn Problem

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Let $\mathcal{H}$ be a separable Hilbert space and K-g-frames as a generalization of g-frames. Indeed, Suppose that J is a countable subset of natural numbers, $\{\mathcal{H}_j\}_{j\in J}$ is a family of closed subspaces of $\mathcal{H}$ and $\Lambda_j \in \mathcal{B}(\mathcal{H}, \mathcal{H}_j)$ $(j \in J)$. Then the sequence $\{\Lambda_j\}_{j\in J}$ is called a g-frame for $\mathcal{H}$ with respect to $\{\mathcal{H}_j\}_{j\in J}$, if there exist the constants $A, B > 0$ such that

$$A\|f\|^2 \leq \sum_{j=1}^{\infty} \|\Lambda_j(f)\|^2 \leq B\|f\|^2 \qquad (f \in \mathcal{H}).$$

Furthermore, $\{\Lambda_j\}_{j\in J}$ is called a K-g-frame for $\mathcal{H}$ with respect to $\{\mathcal{H}_j\}_{j\in J}$, if there exist constants $A, B > 0$ such that

$$A\|K^*f\|^2 \leq \sum_{j=1}^{\infty} \|\Lambda_j(f)\|^2 \leq B\|f\|^2 \quad (f \in \mathcal{H}).$$

In this study [1], we identify gaps in existing proofs concerning K-g-frames and present examples showing these results are not necessarily valid. We then address these gaps and provide revised conclusions. Specifically, we explore the Schur-Horn problem, characterizing sequences $\{\|f_n\|^2\}_{n=1}^{\infty}$ for all frames $\{f_n\}_{n=1}^{\infty}$ with the same frame operator. We introduce the concept of synthesis-related frames and investigate the Schur-Horn problem for finite-dimensional $\mathcal{H}$, proving that two frames have the same frame operator if and only if they are synthesis-related.

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Strong and Weak Type Estimates for the Littlewood-Paley Operator g^*_λ, ψ with Non-Convolution Kernel

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In this work, the $(1,1)-$ weak type boundedness and $(p,p)-$ strong type boundedness of the Littlewood-Paley operator $g^*_{\lambda,\psi}$ with non-convolution kernel ψ for $\lambda > 2$ have been estimated via the Calderón-Zygmund approach. In particular, we show that the Littlewood-Paley operator $g^*_{\lambda,\psi}$ is a Hilbert-valued Calderón-Zygmund operator. Consequently, by a fundamental theorem in [1,2], one can conclude the $(1,1)-$ weak type and $(p,p)-$ strong type boundedness of the operator $g^*_{\lambda,\psi}$, for $\lambda > 2$, when $1 < p < \infty$. It should be noted that the vector-valued Calderón-Zygmund approach simplifies the proof of the above results.

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Strength Theory of Layered Fibrous Polymer Composites

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When analyzing the strength of structures made of layered fibrous polymer composite materials the criteria for failure of a monolayer – a unidirectional reinforced composite – are used. A criterion of strength according to the conditions of matrix failure is formulated, corresponding to conical limiting surfaces and the lowest loads to failure. The criterion of strength according to the condition of fiber failure, which does not allow the paradox of increasing strength in the region of transition from fiber fracture to matrix fracture, is given. Experimental verification of the criteria for three-dimensional stress, plane stress, and uniaxial stress is carried out. Their better correspondence to empirical data is shown and their advantages in comparison with known criteria are marked. Based on these criteria, an algorithm for ply-by-ply analysis of the composite strength of typical stacks for thin-walled structures is presented. Formulas for the relationship between the strength of the composite package as a whole and monolayers are given. In step-by-step loading, monolayers are sequentially determined, in which the strength limits are achieved under the conditions of matrix fracture or fibers fracture. Further, these layers are considered to be partially or completely destroyed, respectively. Loads

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Analytical Integration in Integral Equation Methods

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In this talk, we introduce explicit formulas for the calculation of matrix entries within the context of Galerkin integral equation methods for the numerical solution of boundary value problems in 3D. The computation involves integrating singular kernel functions over pairs of surface panels, which becomes challenging when these panels intersect. While coordinate transformations can eliminate singularities, the use of numerical integration remains expensive since the integrals are still four-dimensional. Our alternative approach makes use of analytical integration for the standard Galerkin discretization of the Laplace equation, focusing on piece-wise constant and linear boundary elements on flat triangles. We demonstrate that employing the Duffy transformation yields regularized integrals, which admit closed and exact formulas. This method enables the accurate computation of matrix entries while significantly reducing computational costs compared to full numerical integration. We validate the accuracy of the new formulas and showcase their efficiency in numerical experiments.

Self-Improving Property of the Hardy-Littlewood Maximal Operator Over Spaces of Homogeneous Type

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The question we address is, "If the Hardy–Littlewood maximal operator—classically defined for a measurable function f on a quasi-metric measure space (Ω, d, μ) by

$$Mf(x) = \sup_{B \ni x} \frac{1}{\mu(B)} \int_B |f(y)|\, d\mu(y),$$

where the supremum of the integral means of f is taken over all balls B containing a point $x \in \Omega$—is bounded on a certain function space, can we automatically extend the boundedness to a family of closely related spaces?" When the answer is yes, such a property of M is referred to as the self-improving property.

We prove the self-improving boundedness of the maximal operator on quasi-Banach lattices with the Fatou property in the setting of spaces of homogeneous type. Our result is a generalization of the boundedness criterion obtained in 2010 by Lerner and Ombrosi for maximal operators on quasi-Banach function spaces over Euclidean spaces [1]. The speciality of the proof for spaces of homogeneous type lies in using adjacent grids of Hytönen–Kairema dyadic cubes [2] and studying the operator M alongside its "dyadic" version. Then we apply the obtained result to variable Lebesgue spaces over spaces of homogeneous type.

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Time-Fractional Diffusion Equation

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The main goal is to study the following time-fractional diffusion equation

$$D^{\alpha}_{a+,t} u = b(t)\Delta_x u + c(t,x)u + f(t,x),\ (t,x) \in (a,T] \times \Omega := Q,$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with regular boundary $\partial\Omega$, with the Dirichlet boundary condition

$$u(t,x)=0,\ t>a,\ x\in\partial\Omega,$$

or the Neumann boundary condition

$$\frac{\partial u}{\partial \eta}=0,\ t>a,\ x\in\partial\Omega,$$

where η is the outward normal and the initial condition

$$\lim_{t\to a}\Gamma(\alpha)\left(\log\frac{t}{a}\right)^{1-\alpha}u(a,x)=u_0(x).$$

A-priori decay estimates of the solution have been studied. To calculate the diffusion equations we use Leibniz's Rule for fractional derivatives.

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Approximation by Matrix Transform Means with Respect to the Vilenkin System in Lebesgue Spaces

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Unlike the classical theory of the Fourier series which deals with the decomposition of a function into sinusoidal waves the Vilenkin (Walsh) functions are rectangular waves. Some important steps in early development can be found in the book by F. Schipp, W. R. Wade, P. Simon and J. Pál [7] from 1990. The research continued intensively also after this. The recent book by L. E. Persson, G. Tephnadze and F. Weisz [6] from 2022 presents some of the most important steps in these developments.

Summability of various means of Fourier series with respect to classical orthonormal systems of integrable functions has a great history. The results obtained in this direction essentially determine the problems in Function Theory and Harmonic Analysis. Matrix transform means are common generalizations of several well-known summation methods. It follows by simple consideration that the Nörlund and weighted T means, Fejér and (C,α) means are special cases of the matrix transform summation method.

This talk is devoted to improving and complementing results proved in [1], [2], [4] and [5]. In particular, we demonstrate more general approximation results for matrix transform means with respect to the Vilenkin system in Lebesgue spaces for any $1\leq p<\infty$ (for details see [3]).

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Marcinkiewicz–Zygmund Inequalities in (Quasi-)Banach Spaces

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The classical MZ inequality states that

$$C^{-1}\Big(\frac{1}{2n+1}\sum_{k=0}^{2n}|T(t_k)|^p\Big)^{1/p}\le\|T\|_{L_p(\mathbb{T})}\le C\Big(\frac{1}{2n+1}\sum_{k=0}^{2n}|T(t_k)|^p\Big)^{1/p},\qquad 1<p<\infty,$$

for any trigonometric polynomial

$$T(x)=\sum_{k=-n}^{n}c_ke^{ikx},\quad c_k\in\mathbb{C},$$

and

$$t_k=\frac{2\pi k}{2n+1},\quad k=0,\dots,2n.$$

We discuss how to obtain analogues of MZ inequalities in general (quasi-)Banach spaces.

Transmission Problems for a Second Order Differential Equation on a Hypersurface with Lipschitz Boundary in the Generic Bessel Potential Spaces

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We are given a hypersurface $\mathcal{C}\subset\mathbb{R}^3$ with the Lipschitz boundary $\Gamma:=\partial\mathcal{C}$, containing angular points $c_1,\dots,c_n$. The surface is divided by a finite number of curves $\mathcal{T}_1,\dots,\mathcal{T}_m$ in non-intersecting domains $\mathcal{C}_1,\dots,\mathcal{C}_{m+1}$ and in each domain $\mathcal{C}_k$ is given Laplace-Beltrami equation with lower order perturbations $\Delta_{\mathcal{C}}u+\mathbf{P}_k(\mathcal{D})u=f_k$, $k=1,\dots,m+1$. The Dirichlet, Neumann and mixed type BVPs are considered on the outer boundary Γ, while on curves $\mathcal{T}_1,\dots,\mathcal{T}_m$ are prescribed transmission conditions. The BVP is treated in a non-classical setting, when solution u is sought in the generic Bessel potential space with exponential weights $r_{\mathcal{C}_\|}u\in\mathbb{GH}^s_p(\mathcal{C}_k,\rho)$, $s>1/p$, $1<p<\infty$, $\rho(t):=\prod_{j=1}^{n}|t-c_j|^{\beta_j}$, $k=1,\dots,m+1$. First, we get rid of transmission conditions and transmission curves and reduce the problem to the Fredholm equivalent Boundary Integral Equation (BIE) on the boundary of the surface $\Gamma=\partial\mathcal{C}$. Second, we apply the localization and reduce the obtained BIE to the investigation of the Model BIE corresponding to Dirichlet, Neumann and mixed BVPs for the Laplace equation in planar angular domains $\Omega_{\alpha_j} \subset \mathbb{R}^2$, $j = 1, 2, \dots, n$, associated to the angular points $c_1, c_2, \dots, c_n$. Third is investigated the model BIE in the generic Bessel potential spaces with weight $\mathbb{GH}^s_p(\Omega_{\alpha_j}, t^{\beta_j})$. For this we reduce further the BIE to a Fredholm equivalent Mellin convolution integral equations in the generic Bessel potential spaces on semi-infinite axes with weight $\mathbb{GH}^{s-1/p}_p(\mathbb{R}^+, t^{\beta_j})$. Explicit criteria of Fredholm property of the initial BVPs are obtained. In contrast to the same BVPs in the classical Bessel potential spaces $\mathbb{H}^s_p(\mathcal{C})$, the Fredholm property in the generic Bessel potential spaces $\mathbb{GH}^s_p(\mathcal{C}, \rho)$ with weight is independent of the smoothness parameter s. We also list explicit singularities of solutions to the mixed-transmission BVP in the neighbourhood of knots, where the boundary has angular points or Dirichlet-Neumann boundary conditions collide.

Numerical Treatment for the Nonlinear Dynamic String Equation of Kirchhoff-Type with Time-Dependent Coefficients

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We consider the nonlinear dynamic string equation of Kirchhoff-type with time-dependent coefficients, a hyperbolic partial differential equation that accurately models the behaviour of an elastic string of length ℓ (see [1] for details). Allowing the material parameters to become time-dependent enriches the dynamics and paves the way for numerous important applications. Oscillating material parameters can model vibrating systems where the vibration influences material properties over time. Additionally, time-varying material constants allow for the modelling of environmental variations such as temperature, pressure, or humidity.

The objective of this study is to develop a time-domain discretization algorithm capable of approximating solutions to this initial-boundary value problem. To this end, a symmetric three-layer semi-discrete scheme is employed with respect to the temporal variable, wherein the nonlinear term is evaluated at the midpoint node. This approach allows for the numerical solutions at each temporal step to be obtained by inverting linear operators, resulting in a system of second-order linear ordinary differential equations. The local convergence of the proposed scheme is established, revealing quadratic convergence concerning the step size of the temporal discretization on the local interval.

From a numerical implementation perspective, a fourth-order accuracy finite-difference scheme is developed for the spatial variable, yielding a tridiagonal system of linear equations at each time step. The coefficient matrices of these systems exhibit strict diagonal dominance. Estimates of the condition numbers of these coefficient matrices are provided, establishing a relationship between the spatial and temporal grid lengths that ensures the condition numbers remain within an acceptable range. The stability of the resulting three-point system from the spatial discretization algorithm is also demonstrated.

References

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On the Existence of the Solution to the Goursat Problem for a Loaded System of Hyperbolic Equations

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In the region Ω, we consider the Goursat problem for a system of loaded differential equations of hyperbolic type second-order

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u}{\partial x} + B(x,t)\frac{\partial u}{\partial t} + C(x,t)u + C_0(x,t)u\,(x_0,t) + f(x,t)$$

$$u(0,t) = \psi(t), \quad t \in [0,T]$$

$$u(x,0) = \varphi(x), \quad x \in [0,\omega]$$

Various problems for loaded hyperbolic methods were studied in [1-3], where you can also find a review and bibliography graphics on agreed topics.

Acknowledgments
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Alphabetical Index of authors

Useful Information

If you require a **taxi**, please be aware that **taxis** in Tbilisi are typically unmetered, and as a result, tourists can occasionally fall victim to overcharging. This makes hailing taxis from the street a somewhat risky endeavour. We highly recommend downloading a ride-hailing app such as **Bolt** and using it to book your taxis in Tbilisi. A journey between the airport and the city centre, or vice versa, when booked through **Bolt**, should commonly not exceed ***15 Euros*** in cost.

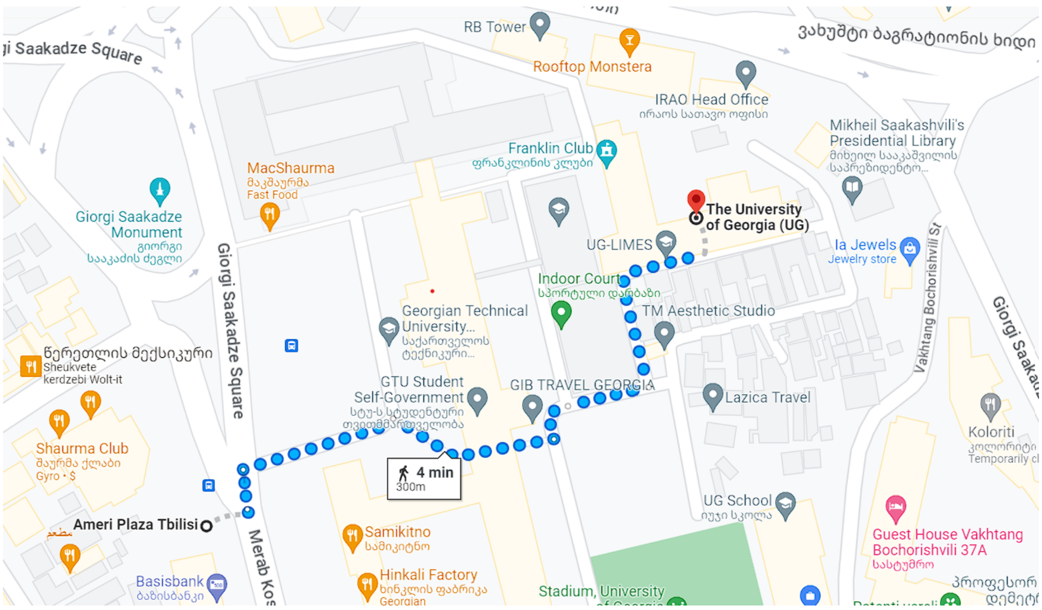
In the public transport of Tbilisi, passengers should use **Visa**, **MasterCard**, or **Amex** cards. It is possible to use **Google Pay** or **Apple Pay** as well. It is mandatory to tap the card on the payment box located on the bus or mini-bus as soon as passengers board the transport. Please be aware that passengers without a ticket will be fined by ticket controllers. When travelling with city transport, plan your route using **Google Maps**, or download the ***Tbilisi Transport*** mobile app, available for both **Android** and **iOS**. For detailed and useful information about public transportation in Tbilisi, you can visit the following link: `https://ttc.com.ge/en`.

For lunch, we recommend the restaurants **"Samikitno-Matchakhela"** and **"Khinkali Factory"**, which are located on the opposite side of the **Ameri Plaza Hotel Tbilisi** in close proximity to the **University of Georgia (UG)**. You can find them at the following address: **77 Merab Kostava St., Tbilisi 0171**. If you prefer a food delivery service to your location, Tbilisi offers several international food delivery apps, such as **Glovo**, **Wolt**, and **Bolt Food**.

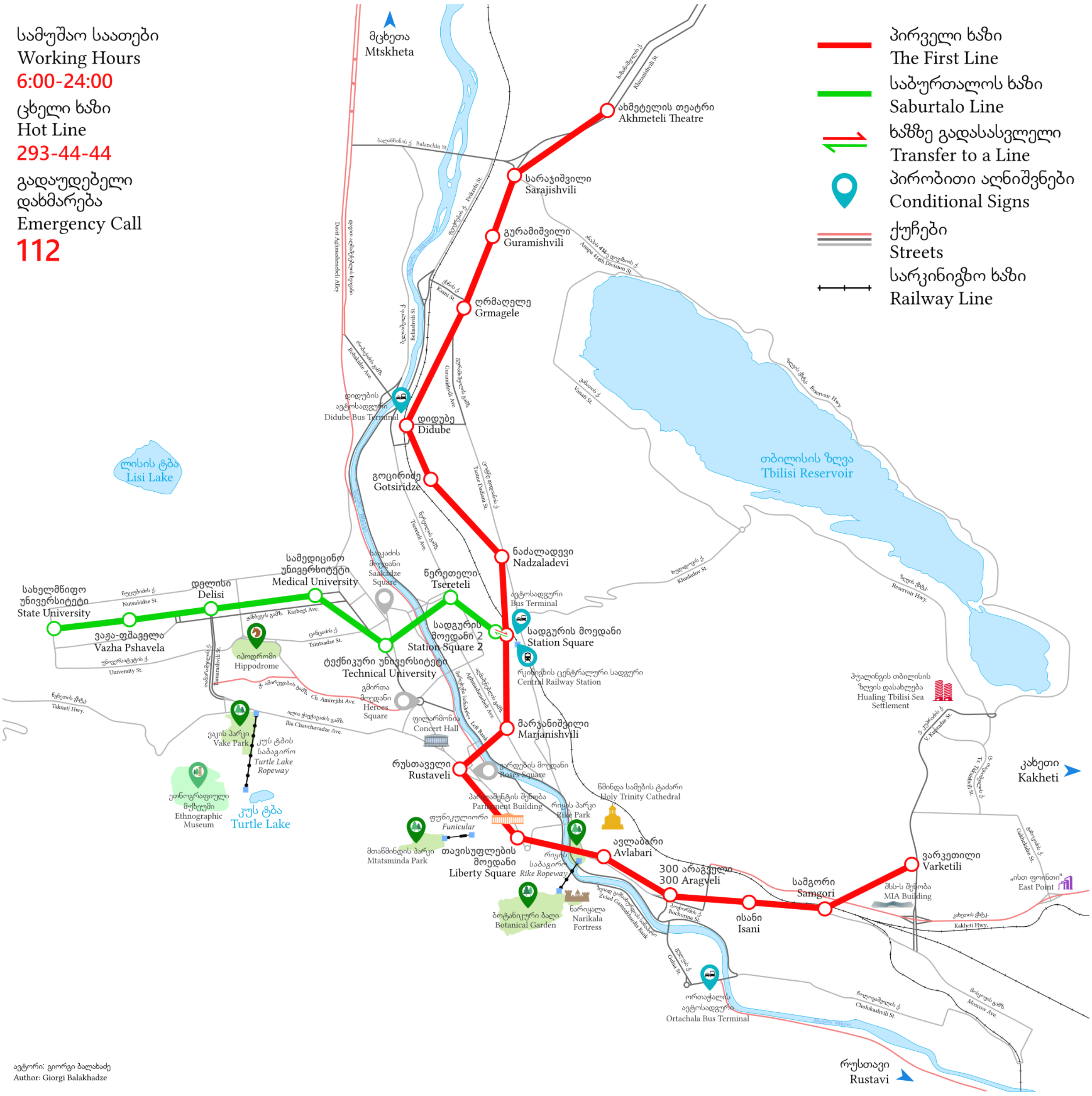
Like many countries, most restaurants, supermarkets, and other establishments in our region accept **credit** or **debit card** payments (**Visa**, **MasterCard**, or **Amex**). However, if you prefer to use **cash**, we recommend exchanging currency on **Pekini Ave.**, which is conveniently located near the **conference venue**. Please be aware that exchanging currency at **banks**, **airports** and in **touristic zones** (such as the Liberty Square neighbourhood) might not provide favourable exchange rates for you.

The front (main) entrance of Hotel **Ameri Plaza**, located at Saakadze Square, faces a bus lane where taxis are not permitted to stop or park. To ensure a convenient pickup, it is recommended to book a taxi to the following address: **9 Ipolite Khvichia Street, Tbilisi, Georgia** (see Google Maps: `https://maps.app.goo.gl/rSYKyCoKKUxxonqP6`). This location is parallel to **Kostava Street**, situated behind **Saakadze Square**, and serves as the *back entrance* to **Ameri Plaza**. Upon arrival at the *back entrance*, you will notice a large inscription reading **"Ameri Plaza"** on your left side.

The route from the **Ameri Plaza Hotel Tbilisi** to the conference venue, **the University of Georgia (Tbilisi)**, takes approximately **4-5 minutes** on foot and covers a distance of about **300 meters**. You can view the route on Google Maps by following this link: `https://goo.gl/maps/bZswfPAa7oxJTfi99`.



The closest underground station to the conference venue is the **Technical University** station. See the map below



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