TAPDE 2025

Program & Abstracts
Tbilisi Analysis & PDE Workshop

Dedicated to the 80th Anniversary of

Professor Roland Duduchava

August 27-30, 2025

V. Kupradze Institute of Mathematics
University of Georgia, Tbilisi

The book of abstracts has been assembled by *Zurab Vashakidze*
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The digital edition of this booklet is accessible through the following link:
https://tapde-workshop.ug.edu.ge/

The source of this template can be traced back to LaTeXTemplates.com, and it has been developed based on the initial version accessible at:
https://github.com/maximelucas/AMCOS_booklet

The organizers of the **TAPDE workshop** wish to express their sincere gratitude to **Arne Hendrickx** for his assistance in designing the cover for this book of abstracts and the poster for the workshop, with the aid of the *Artificial Intelligence (AI)* tool *Microsoft Copilot*.

The most recent update of the **Program** and the **Book of Abstracts** was completed on **August 26, 2025**.

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About

This is a generic version of the real AMCOS conference booklet for which this LaTeX template was generated. All information about the use and distribution of this template, and all related codes, can be found at `https://github.com/maximelucas/AMCOS_booklet`.

TAPDE 2025

The Tbilisi Analysis & PDE Seminar (TAPDE) was established in November 2020 as a bi-weekly series of online seminars, featuring presentations from over 70 distinguished scholars from various countries. Although the initial plan included complementing the online talks with an annual workshop, this was postponed due to the COVID-19 pandemic until 2023, when the first **Tbilisi Analysis & PDE Workshop** (TAPDE2023)[1] was held in Tbilisi, the capital of Georgia. This workshop has now become an annual international event.

The workshop covers a broad range of topics, including Real and Complex analysis, Operator Theory, Harmonic Analysis, Integral Equations, Numerical Analysis, Partial Differential Equations, Mathematical Physics, and related fields. We invite researchers to submit their work for presentation, share their latest research findings, and engage in discussions about new developments in their respective areas of expertise.

Committees

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Editors: Roland Duduchava, Eugene Shargorodsky, Emzar Khmaladze, Zurab Vashakidze.

[1] https://tapde-workshop.ug.edu.ge/2023/

Professor Roland Duduchava — 80th Anniversary

This year marks the **80th anniversary** of Professor Roland Duduchava, an eminent Georgian mathematician whose contributions to the theory of integral and partial differential equations are recognized worldwide. He is the author and co-author of five monographs and 134 research papers. His results are widely used by researchers working on singular integral equations, pseudodifferential equations, boundary value problems for elliptic partial differential equations, and many other areas of pure mathematics and applications.

Roland Duduchava was born on November 12, 1945, in Tbilisi. He graduated from a secondary school in Sokhumi (Georgia) in 1962 and enrolled in the Faculty of Mechanics and Mathematics of Tbilisi State University, from which he graduated with honors in January 1968. He then became a Ph.D. student at the A. Razmadze Mathematical Institute of the Georgian Academy of Sciences, Tbilisi, Georgia. In 1971, he completed his Ph.D. studies in Kishinev, Moldova, at the Institute of Mathematics and Computing Center of the Academy of Sciences of Moldova, and in the same year defended his Candidate Thesis (Ph.D. degree) under the supervision of Professor Israel Gohberg.

Since 1971, Roland Duduchava has served as a junior, senior, leading, and principal researcher at the A. Razmadze Mathematical Institute of the Georgian Academy of Sciences, and since 1995, he has been Head of the Department of Mathematical Physics. Since 2016, he has been a professor at the University of Georgia (Tbilisi) and, since 2018, the Director of the Victor Kupradze Institute of Mathematics at the same university. He also supervises the Ph.D. programme in mathematics there.

In 1983, he defended his higher doctoral thesis (Habilitation) at M. Lomonosov Moscow State University. In 1989, he was granted the title of Professor by the Supreme Attestation Commission of the USSR.

At various times, Roland Duduchava has worked as a professor at I. Javakhishvili Tbilisi State University, IB Euro-Caucasian University, Humboldt University in Berlin, Saarland University in Saarbrücken, and Stuttgart University.

Roland Duduchava is a fellow of the Alexander von Humboldt Foundation (1981–1989) and a Professor Mercator of the German Research Council (DFG) (2001–2002).

He has received ten international (Soros, AMS, INTAS, DFG, and others) and four national (GNSF – Shota Rustaveli National Science Foundation) research grants as the head of a research group.

Roland Duduchava serves on the editorial boards of six international mathematical journals. He has successfully supervised eight Ph.D. students and has acted as a consultant for one higher doctoral thesis and for two Ph.D. students in Germany.

From 2007 to 2011 and from 2009 to 2017, he served as President of the Georgian Mathematical Union and organized many international conferences, including the International Conference "Continuum Mechanics and Related Problems of Analysis" dedicated to the 120th birthday of Academician N. Muskhelishvili (2011), the Caucasian Mathematics Conference (2014), the International Workshop on Operator Theory and Applications (IWOTA 2015), and others. Roland Duduchava is the founder and chairman of the first ten annual international conferences in Batumi, as well as the Tbilisi Analysis and PDE Workshop (TAPDE).

Professor Roland Duduchava is an outstanding researcher whose life is full of great achievements in mathematics.

We warmly congratulate Professor Roland Duduchava on this occasion and wish him good health, happiness, and continued success in the years ahead.

The organizing committee extends its gratitude to David Kapanadze and David Natroshvili for their time and effort in preparing the short biography of Professor Roland Duduchava.

Timetable

Invited Talks (IT): The invited talks are scheduled to last **60 minutes**, with **5 minutes** included for questions.
Contributed Talks (CT): The contributed talks are scheduled for **30 minutes**, including **5 minutes** for questions.

August 27, Wednesday, Session at the University of Georgia, Tbilisi

09:00–09:30	**Registration, Room 519**		
09:30–10:00	**Opening, Room 519**		
Room 519	**Plenary Session**		
Chairman	**Eugene Shargorodsky**		
10:00–11:00	IT	**Roland Duduchava** Tbilisi, Georgia	Fractional Differential Equations on Lie Groups and Submonoids
11:00–11:30	**Coffee Break**		
Room 305	**Plenary Session**		
Chairman	**Guillermo P. Curbera**		
11:30–12:30	IT	**Michael Ruzhansky** Ghent, Belgium	Fujita Exponent for Hörmander Vector Fields
12:30–13:30	IT	**Lars-Erik Persson** Narvik, Norway; and Uppsala, Sweden	Old and New on Hardy-Type Inequalities and Convexity
Room 318	**Plenary Session**		
Chairman	**Grigori Rozenblum**		
11:30–12:30	IT	**Andrey V. Shanin** Moscow, Russia	Multidimensional Complex Analysis for the Problems of Wave Radiation and Scattering
12:30–13:30	IT	**Valery P. Smyshlyaev** London, United Kingdom	Multiscale Approximations for Asymptotically Degenerating Operators and Their Spectra
13:30–14:30	**Lunch Break**		
Room 305	**Session: Differential Equations and Applications**		
Chairman	**David Natroshvili**		
14:30–15:00	CT	**Otar Chkadua** Tbilisi, Georgia	Dynamic Nonclassical Transmission Problems of Generalized Thermo-Electro-Magneto-Elasticity
15:00–15:30	CT	**Tengiz Buchukuri** Tbilisi, Georgia	Pseudo-Oscillation Equation of Heat Transfer in Thin Plates
15:30–16:00	CT	**Tatiana Kuzmina** Tbilisi, Georgia	Equation of State for a Composite at Crack Tip and Displacement Discontinuity Intensity Factor
16:00–16:30	**Coffee Break**		
Chairman	**Duván Cardona**		
16:30–17:00	CT	**Daulet Nurakhmetov** Almaty and Astana, Kazakhstan	On the Behavior of Vibrations of Non-Uniform Beams

17:00–17:30	CT	**Zahra Keyshams** Yerevan, Armenia	Very Weak Solutions of the Heat Equation with Anisotropically Singular Time-Dependent Diffusivity
17:30–18:00	CT	**Madi Yergaliyev** Almaty, Kazakhstan	On Solvability of Strongly Degenerate Hyperbolic Equations
Room 318	**Session: Algebra, Topology, and Applications**		
Chairman	**Teimuraz Pirashvili**		
14:30–15:00	CT	**Bachuki Mesablishvili** Tbilisi, Georgia	On Factorization of Groups, Monoids and Skew Braces
15:00–15:30	CT	**Dali Zangurashvili** Tbilisi, Georgia	Another Characterization of Left Hereditary Left Perfect Right Coherent Rings
15:30–16:00	CT	**Ilia Pirashvili** Loerrach, Germany	Homotopy in the Geometry of Monoids
16:00–16:30	**Coffee Break**		
16:30–17:00	CT	**Anzor Beridze** Batumi and Kutaisi, Georgia	The Braid Group, Garside Structure and Burau Representation
17:00–17:30	CT	**Malkhaz Bakuradze** Tbilisi, Georgia	Equivariant Embeddings of Trivial G-Bundles Into Complex G-Bundles Over Finite G-CW Complexes
18:00	**Welcome Party – Room 222**		

August 28, Thursday, Session at the University of Georgia, Tbilisi

09:30–10:00	**Registration, Room 222**		
Room 305	**Plenary Session**		
Chairman	**Jani Virtanen**		
10:00–11:00	IT	**Alexander Meskhi** Tbilisi and Kutaisi, Georgia	Trace Inequality for Fractional Integrals, and Related Topics
11:00–11:30	**Coffee Break**		
11:30–12:30	IT	**Leonid Parnovski** London, United Kingdom	Classical Spectral Asymptotics with a Modern Twist
12:30–13:30	IT	**Clifford Gilmore** Clermont-Ferrand, France	Universality of Composition Operators and Applications to Complex Dynamics
Room 318	**Plenary Session**		
Chairman	**Malte Braack**		
10:00–11:00	IT	**Grigori Rozenblum** Gothenburg, Sweden	Eigenvalues Properties of the Neumann-Poincaré Operator in 3D Elasticity
11:00–11:30	**Coffee Break**		
11:30–12:30	IT	**David Natroshvili** Tbilisi, Georgia	Application of the Potential Method to Mixed Boundary Value Problems for Viscoelastic Solids with Voids
12:30–13:30	IT	**Berikbol T. Torebek** Almaty, Kazakhstan	Fujita-Type Critical Exponents for the Semilinear Parabolic Problems
13:30–14:30	**Lunch Break**		
Room 305	**Session: Real and Complex Analysis**		

Chairman	Lasha Ephremidze		
14:30–15:00	CT	**Alina Shalukhina** Caparica, Portugal	Interpolation of Variable Lebesgue Spaces Over Spaces of Homogeneous Type
15:00–15:30	CT	**Giorgi Tutberidze** Tbilisi, Georgia	Generalized Fourier-Cesàro Sums for Functions of Bounded Variation
15:30–16:00	CT	**Haakan Hedenmalm** St. Petersburg, Russia	Bombieri-Type Inequalities and Equidistribution of Points
16:00–16:30	**Coffee Break**		
Chairman	**Haakan Hedenmalm**		
16:30–17:00	CT	**Lasha Ephremidze** Tbilisi and Kutaisi, Georgia	Matrix Spectral Factorization Algorithm with Non-Commutative Coefficients
Room 318	**Session: Algebra, Topology, and Applications**		
Chairman	**Bachuki Mesablishvili**		
14:30–15:00	CT	**Teimuraz Pirashvili** Tbilisi, Georgia	On k-Invariants of Crossed Modules Related to Dihedral Groups
15:00–15:30	CT	**Emzar Khmaladze** Tbilisi, Georgia	Relations Between Categories of Crossed Modules of Various Algebras
15:30–16:00	CT	**Mariam Pirashvili** Plymouth, United Kingdom	An Isometry Theorem for Persistent Homology of Circle-Valued Functions
16:00–16:30	**Coffee Break**		
16:30–17:00	CT	**Giorgi Rakviashvili** Tbilisi, Georgia	On Projective Class Groups of Crossed Group Rings
17:00–17:30	CT	**Tsotne Pavliashvili** Tbilisi, Georgia	Extremal Problems on Four Concentric Circles
19:00	**Conference Dinner at the Restaurant "Ethnographer"**		

The restaurant **"Ethnographer"** is located at **105 Akaki Beliashvili Street, Tbilisi 0159, Georgia**. It can be found on Google Maps via the following link: https://maps.app.goo.gl/zJSEBt8xgK9atc4p6. Please note that the cost of dinner at the restaurant *per person* is included in the conference fee of **USD 100**.

August 29, Friday, Day off, Excursion

09:00	**Excursion to Kazbegi, with Lunch** **Departure from the front entrance of the University of Georgia, Tbilisi**

The conference fee of **USD 100** covers the **excursion with lunch** for each participant.

Kazbegi Municipality is a mountainous region in northeastern **Georgia**, located within the **Mtskheta–Mtianeti** area. It is renowned for its dramatic landscapes, alpine meadows, and historic landmarks such as the 14th-century *Gergeti Trinity Church*. Its most prominent natural feature is **Mount Kazbek** (*Mkinvartsveri* in Georgian), a dormant stratovolcano that rises to **5,054 m** (**16,581 ft**) above sea level, making it one of the highest peaks in the **Caucasus**. **Mount Kazbek** is the highest mountain in **Eastern Georgia** and the third-highest peak in the country, after **Mount Shkhara** and **Janga**. The area is part of *Kazbegi National Park*, a popular destination for hiking, mountaineering, and cultural tourism.

August 30, Saturday, Session at the University of Georgia, Tbilisi

Room 305	**Plenary Session**		
Chairman	**Clifford Gilmore**		
10:00–11:00	IT	**David Cruz-Uribe, OFS** Tuscaloosa, Alabama, USA	Recent Results on Matrix Weighted Norm Inequalities
11:00–12:00	IT	**Duván Cardona** Ghent, Belgium	Spectral and Analytic Properties of Fourier Integral Operators with Complex Phases
12:00–12:30	**Coffee Break**		
12:30–13:30	IT	**Guillermo P. Curbera** Sevilla, Spain	The Unbounded Finite Hilbert Transform
Room 318	**Plenary Session**		
Chairman	**Valery P. Smyshlyaev**		
10:00–11:00	IT	**Marianna Chatzakou** Ghent, Belgium	On a Class of Anharmonic Oscillators
11:00–12:00	IT	**Malte Braack** Kiel, Germany	Mapped Coercive Nonlinear Operators and Applications to Saddle-Point Problems
12:00–12:30	**Coffee Break**		
12:30–13:30	IT	**Jani Virtanen** Helsinki and Joensuu, Finland; and Reading, United Kingdom	The Hilbert Matrix and Its Generalizations
13:30–13:45	**Closing**		
13:45–14:30	**Lunch Break**		
Room 305	**Session: Differential Equations and Applications**		
Chairman	**Leonid Parnovski**		
14:30–15:00	CT	**Bibigul Omarova** Aktobe, Kazakhstan	Application of the Method of Helical Characteristics to the Study of Multiperiodic Solutions of Lyapunov's D-Systems
15:00–15:30	CT	**Zhaishylyk Sartabanov** Aktobe, Kazakhstan	The Method of Helical Characteristics and Its Application to Problems in the Theory of Multifrequency Oscillations
15:00–15:30	CT	**Jemal Rogava, Zurab Vashakidze** Tbilisi, Georgia	On the Approximate Solution of the Initial–Boundary Value Problem for Nonlinear Kirchhoff-Type Equations with Time-Dependent Material Coefficients
16:00–16:30	**Coffee Break**		
Chairman	**Tengiz Buchukuri**		
16:30–17:00	CT	**Monire Mikaeili Nia** Yerevan, Armenia	Existence and Uniqueness Theorems for One Class of Hammerstein-Type Nonlinear Integral Equations
17:00–17:30	CT	**Besiki Tabatadze** Tbilisi, Georgia	Deep Neural Network Approach for Solving a Two-Dimensional Nonlinear Model

List of Abstracts – Talks

Invited Talks (IT)

Mapped Coercive Nonlinear Operators and Applications to Saddle-Point Problems

Malte Braack[1], ***Roland Becker***[2]

IT

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In this talk we address the challenge of proving the existence of solutions for nonlinear equations in Banach spaces, focusing on the Navier–Stokes equations and their discretizations. Traditional methods, such as monotonicity-based approaches and fixed-point theorems, often face limitations in handling general nonlinear operators or finite element discretizations. A novel concept, mapped coercivity, provides a unifying framework to analyze nonlinear operators through a continuous mapping. We apply these ideas to saddle-point problems in Banach spaces, emphasizing both infinite-dimensional formulations and finite element discretizations. Our analysis includes stabilization techniques to restore coercivity in finite-dimensional settings, ensuring stability and existence of solutions. For linear problems, we explore the relationship between the inf-sup condition and mapped coercivity, using the Stokes equation as a case study. For nonlinear saddle-point systems, we extend the framework to mapped coercivity via surjective mappings, enabling concise proofs of existence of solutions for various stabilized Navier–Stokes finite element methods, including Brezzi–Pitkäranta, a simple variant, and local projection stabilization techniques, with extensions to convection-dominant flows. The proposed methodology offers a robust tool for analyzing nonlinear PDEs and their discretizations, bypassing traditional decompositions and providing a foundation for future developments in computational fluid dynamics.

References

[1] R. Becker, M. Braack. *Mapped coercivity for the stationary Navier–Stokes equations and their finite element approximations*. To appear 2025 in CMAM.

[2] R. Becker, M. Braack. *The concept of mapped coercivity for nonlinear operators in Banach spaces*. J. Funct. Anal. 289 (3), 110893, 2025. `https://doi.org/10.1016/j.jfa.2025.110893`

Spectral and Analytic Properties of Fourier Integral Operators with Complex Phases

Duván Cardona

IT

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In this talk we present some results about the nuclearity of Fourier integral operators with complex phases.

References

[1] D. Cardona, J. Delgado, M. Ruzhansky. *Nuclearity, Schatten–von Neumann classes, distribution of eigen-*

values and L^p–L^q-boundedness of Fourier integral operators on compact manifolds, submitted. arXiv:2408.06833.
[2] D. Cardona, M. Ruzhansky. *The weak $(1,1)$ boundedness of Fourier integral operators with complex phases*, submitted. arXiv:2402.09054.

On a Class of Anharmonic Oscillators

Marianna Chatzakou[1], ***Julio Delgado***[2], ***Michael Ruzhansky***[1,3]

IT

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[2] Departamento de Matemáticas, Universidad del Valle, Colombia
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We will discuss a class of anharmonic oscillators on $\mathbb{R}^n$ corresponding to Hamiltonians of the form $A(D)+V(x)$, where $A(\xi)$ and $V(x)$ are smooth functions satisfying certain regularity conditions. Our analysis is in the framework of Weyl-Hörmander classes of operators. The relation of the latter with the Schatten–von Neumann classes of operators will be used to derive spectral properties of these operators. The talk is based on the works [1, 2].

References

[1] M. Chatzakou, J. Delgado, and M. Ruzhansky. On a class of anharmonic oscillators. *J. Math. Pures Appl.*, 153(9) (2021), 1–29.
[2] M. Chatzakou, J. Delgado, and M. Ruzhansky. On a class of anharmonic oscillators II. General case. *Bull. Sci. Math.*, 80 (2022), No. 103196, 22 pp.

Recent Results on Matrix Weighted Norm Inequalities

David Cruz-Uribe, OFS

IT

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Matrix weighted inequalities for singular integrals and other operators have been of interest since the work of Nazarov, Treil and Volberg in the 1990s. They asked whether the classical Muckenhoupt A_p condition could be generalized to matrix weights so that inequalities of the form

$$\int_{\mathbb{R}^n} |W(x)Tf(x)|^p\,dx \le C\int_{\mathbb{R}^n} |W(x)f(x)|^p\,dx,$$

hold for $1<p<\infty$, where W is a $d\times d$ self-adjoint, positive semi-definite matrix, f is an $\mathbb{R}^d$-valued measurable function, and T is a singular integral or other linear operator from classical harmonic analysis. NTV proved bounds for the Hilbert transform on the real line, introducing a generalization of A_p in terms of norm functions on $\mathbb{R}^d$. Their results were extended to $\mathbb{R}^n$ and general Calderón–Zygmund singular integrals by Christ and Goldberg. Later, Roudenko introduced an equivalent definition of matrix $\mathcal{A}_p$ that more closely resembles the classical scalar condition:

$$[W]_{\mathcal{A}_p} = \sup_Q \left(\fint_Q \left(\fint_Q |W(x)W^{-1}(y)|_{\text{op}}^{p'}\,dy \right)^{\frac{p}{p'}} dx \right)^{\frac{1}{p}} < \infty.$$

In the past 10 years there has been a great deal of research in the study of matrix weights. In this talk we discuss recent work in this area, including: Rubio de Francia extrapolation and the machinery of convex set-valued functions developed to prove this result; multiplier weak-type norm inequalities; and extensions to the variable Lebesgue spaces.

This talk includes joint work with Marcin Bownik, Josh Isralowitz, and Kabe Moen, and with my PhD student Michael Penrod and my postdoc Fatih Şirin.

The Unbounded Finite Hilbert Transform

Guillermo P. Curbera[1], ***Susumu Okada***[2], ***Werner J. Ricker***[3] IT

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[2] Kambah, Australian Capital Territory, Australia

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We investigate the unbounded operator obtained from the finite Hilbert transform when it acts from L^1 into itself. A full inversion theorem is established for this operator, together with suitable extended versions of the Parseval and Poincaré–Bertrand formulae.

References

[1] G. P. Curbera, S. Okada, W. J. Ricker. *Inversion of the unbounded finite Hilbert transform on* L^1. Preprint.
[2] G. P. Curbera, S. Okada, W. J. Ricker. *The finite Hilbert transform acting on the Zygmund space* $L\log L$. Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) **25** (2024), 1527–1557.
[3] G. P. Curbera, S. Okada, W. J. Ricker. *The finite Hilbert transform on* $(-1, 1)$. Theta Ser. Adv. Math. **27**, Theta, Bucharest, 2024, 29–60.

Fractional Differential Equations on Lie Groups and Submonoids

Roland Duduchava[1], ***Asselya Smadiyeva***[2] IT

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[2] Institute of Mathematics and Mathematical Modelling, Almaty, Kazakhstan

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Fractional differential operators (FDO) represent pseudodifferential (convolution) operators of fractional order. We define FDO on Lie groups and study properties of these generic fractional derivatives, indicating criteria of unique solvability of equations with fractional derivatives and constant coefficients. Moreover, we investigate equations with fractional derivatives and constant coefficients restricted to submonoids, where the theory of equations becomes much richer (example: Wiener–Hopf equation on the half-axis versus convolution equations on the axis). Criteria of Fredholm property, solvability, and index theory are exposed.

We define and study the modified Hadamard type FDOs $\mathfrak{D}_{\pm}$ on the Lie group $\{\mathbb{R}^+, \times\}$ and the Hadamard type fractional derivatives $\mathfrak{D}_{I,\pm}$ on the Lie group $\{\mathcal{I} = (-1, 1), \mathcal{I}rc\}$. Fundamental solutions to the Hadamard fractional differential operators are given.

We solve six different Cauchy initial-boundary value problems for degenerate and non-degenerate diffusion equations of the following types:

$$\mathfrak{D}^{\alpha}_{+,t}u(t,x) - \lambda(t-a)^{\gamma}\Delta_x u(t,x) = 0$$

on the domain $\mathbb{R}^+_a \times \mathbb{R}^N$, $\mathbb{R}^+_a := (a, \infty)$; and analogous variants on $(a, b] \times \mathbb{R}^N$ or $(a, b] \times \Omega$ with Ω an open, bounded set in $\mathbb{R}^N$ with a smooth boundary $\partial\Omega$, as well as cases with right-hand side $f(t, x)$.

To prove the existence of solutions, we use the Fourier method, spectral properties of the Laplace operator Δ_x on Ω, and basic properties of the Kilbas–Saigo $E_{\alpha,m,l}(z)$ and Mittag–Leffler $E_{\alpha,\beta}(z)$ functions.

The investigation is based on the results obtained in [1,2] and other recent publications.

Acknowledgments. This work was supported by the Horizon 2020 Grant No. 873071 SOMPATY.

References
[1] D. Cardona, R. Duduchava, A. Hendrix, M. Ruzhansky. *Generic Bessel potential spaces on Lie groups*. In: Tbilisi Analysis & PDE Seminar – Extended Abstracts of the 2020–2023 Seminar Talks, Trends in Mathematics, Birkhäuser/Springer, pp. 43–54, 2024.
[2] R. Duduchava. *Convolution equations on the submonoid* **M** $= [0, 1)$. Submitted to Boletín de la Sociedad Matemática Mexicana (BSMM), 37 pages.

Universality of Composition Operators and Applications to Complex Dynamics

Vasiliki Evdoridou[1], ***Clifford Gilmore***[2], ***Myrto Manolaki***[3]

IT

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We study universality properties of composition operators $C_f : g \mapsto g \circ f$, acting on spaces of holomorphic functions, where the symbol f is a transcendental entire function restricted to parts of its Fatou set $F(f)$. Initiated by Jung [2], this line of research lies at the intersection of complex dynamics and operator theory.

We present new results on universality of C_f when the symbol f is restricted to certain Baker and wandering domains, and extend the discussion to weighted composition operators $W_{\omega,f} : g \mapsto \omega \cdot (g \circ f)$. This broader framework permits analogous universality results and further applications to complex dynamics [1].

References
[1] V. Evdoridou, C. Gilmore, M. Manolaki. *Interactions between universal composition operators and complex dynamics*. J. London Math. Soc., in press (2025). arXiv:2409.16260.
[2] A. Jung. *Universality of composition operators and applications to holomorphic dynamics*. J. Anal. Math. **137**:2 (2019), 845–874.

Trace Inequality for Fractional Integrals, and Related Topics

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Trace inequality

$$\|K_\alpha f\|_{L^q_\mu} \le C\|f\|_{L^p}$$

for a fractional integral operator K_α plays an important role in harmonic analysis and PDEs (see, e.g., [1,2]). There are various known types of criteria that ensure this inequality. Our aim is to present recent results regarding transparent necessary and sufficient conditions on a Borel measure μ governing it. The talk consists of the following parts:

1. The problem of finding an appropriate Lorentz space $L^{p,s}$ such that the well-known D. Adams-type condition on μ is both necessary and sufficient for the validity of the trace inequality

$$\|I_\alpha f\|_{L^p_\mu} \le C\|f\|_{L^{p,s}},$$

where I_α is the Riesz potential on $\mathbb{R}^n$. We show that the desired space is $L^{p,1}$. This problem was motivated by the fact that the inequality fails for $p = s$ under the Adams-type condition. The latter condition is necessary and sufficient for the validity of the trace inequality if and only if $1 < p < q < \infty$.

2. A complete characterization of the trace inequality for certain multilinear fractional integral operators T_α:

$$\|T_\alpha(f_1, \dots, f_m)\|_{L^q_\mu} \le C \prod_{k=1}^{m} \|f_k\|_{L^{p_k}}.$$

3. Necessary and sufficient conditions on a (non-doubling) measure μ for which

$$\|J_{\gamma,\mu} f\|_{L^{q,r}_\mu} \le C\|f\|_{L^{p,s}_\mu}$$

holds for a fractional integral operator $J_{\gamma,\mu}$ defined with respect to μ. The classical case for Lebesgue spaces was studied in [3] (see also [4], Ch. 6).

The talk is based on [5–8].

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Application of the Potential Method to Mixed Boundary Value Problems for Viscoelastic Solids with Voids

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We investigate mixed boundary value problems (BVP) of the linear theory of viscoelasticity for isotropic and homogeneous Kelvin–Voigt materials with voids, when on one part of the boundary of the body the Dirichlet-type condition is given, and on the remaining part, the Neumann-type condition is prescribed. Using the potential method and the theory of pseudodifferential equations, we prove the existence and uniqueness of solutions in appropriate Sobolev–Slobodetskii, Bessel potential, and Besov spaces. With the help of embedding theorems, we establish optimal regularity results for solutions to the mixed BVPs near the collision curves where different types of boundary conditions meet. In particular, we prove that the solutions belong to the space of Hölder continuous functions in the closed region occupied by the viscoelastic body. An efficient algebraic algorithm is described for finding the Hölder smoothness exponents, which in turn, determine the corresponding stress singularity exponents near the collision curves. It is shown that these exponents essentially depend on the material parameters.

This is a joint work with Maia Svanadze.

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Classical Spectral Asymptotics with a Modern Twist

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The existence of spectral asymptotics of Laplace or Schrödinger operators acting on Riemannian manifolds is a classical problem known for more than 100 years. It has been known for a long time that obstacles to the existence of spectral asymptotic expansions are periodic and looping trajectories of the geodesic flow. A conjecture formulated in 2016 stated that these trajectories are the only such obstacles.

I will discuss the history of this problem and describe the recent progress: proving this conjecture in special cases, as well as constructing some counterexamples. This is a joint work with Jeff Galkowski (UCL) and Roman Shterenberg (UAB).

Old and New on Hardy-Type Inequalities and Convexity

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Hardy-type inequalities and Convexity (Jensen's inequality) have both more than 100 years of fascinating history in research. In this lecture I present some examples of, in my opinion, especially interesting or important results in these areas and their relation. In particular, some collaboration with Georgian colleagues will be highlighted. More information can be found in the following books (and more recent papers by me and others mentioned in the lecture).

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Eigenvalues Properties of the Neumann-Poincaré Operator in 3D Elasticity

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The Neumann-Poincaré (NP) operator in 3D elasticity for a body with smooth boundary is a singular integral operator acting upon 3-component vector functions on the boundary. It is known to have three points of essential spectrum, 0 and $\pm\mathbb{k}$, where $\mathbb{k}$ is expressed in terms of the Lamé constants, $\mathbb{k} = \frac{\mu}{2(2\mu+\lambda)}$. Thus, the remaining part of the spectrum consists of eigenvalues, possibly converging to the points of the essential spectrum. By means of reduction of the NP operator to a zero order polynomially compact pseudodifferential operator and using the method of investigating the eigenvalue asymptotics of such operators (previously elaborated by the author), we find the asymptotics of series of eigenvalues converging to 0 and $\pm\mathbb{k}$ and determine the dependence of these series on the geometry of the body. In particular, for any body, there are infinite eigenvalue sequences approaching 0 and $\pm\mathbb{k}$ from above. On the other hand, if there exists just one point where the body is strictly concave, there are also infinite eigenvalue sequences converging to 0 and $\pm\mathbb{k}$ from below. This happens, in particular, if the body contains a cavity.

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Fujita Exponent for Hörmander Vector Fields

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In this work, we will review our research on the Fujitsu exponent, in the settings of Lie groups and also for Hörmander's sums of squares.

Multidimensional Complex Analysis for the Problems of Wave Radiation and Scattering

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The talk contains a review of methods of multidimensional complex analysis and analysis on complex manifolds in application to wave-related problems:

1. Solving of transient radiation problems in waveguides. An important function associated with a waveguide is its dispersion diagram, i.e. a set of points in the coordinates (ω, k), where ω is the temporal frequency, and k is the propagation constant, corresponding to propagating modes. It has been shown [1,2] that some benefits can be achieved if a complex domain of (ω, k) is studied; the dispersion diagram is then a 2D complex manifold in this domain.

2. Developing of Sommerfeld integral method on dispersion diagrams. It has been shown that a Sommerfeld integral can be written as a contour integral on a dispersion diagram of the corresponding space. This method has been applied to problems of diffraction of waves in discrete lattices [3–5].

3. Estimation of multidimensional oscillation integrals of Fourier type. In many wave-related problems, one should estimate an integral of the form

$$u(\Lambda) = \int_\Gamma F(\xi) \exp\{i\Lambda G(\xi)\} d\xi_1 \wedge \cdots \wedge d\xi_n,$$

where $F(\xi)$ and $G(\xi)$ are analytic functions in a neighborhood of $\mathbb{R}^n$. The aim is to estimate the integral asymptotically as $\Lambda \to \infty$. An approach similar to the stationary phase method in Poincaré's formulation is developed [6–10].

4. Building of the Wiener–Hopf method in 2D. Two steps have been made in this direction: (i) deriving a new class of integral relations enabling analytic continuation of the solution [11], (ii) proposing a matrix-vector representation of the Picard–Lefschetz formula for studying the evolution of the integral surfaces [12].

The focus of the talk will be on the recent development of these techniques.

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Multiscale Approximations for Asymptotically Degenerating Operators and Their Spectra

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Numerous problems from applications lead to the need for analysing asymptotically degenerating operators and their spectra. A key example is an elliptic operator with high-contrast periodic coefficients in $\mathbb{R}^n$, but there are diverse examples of PDE and other models displaying similar features. This poses a challenge for approximating the related operators as the corresponding limit operators appear to be "two-scale" and so act in different (larger) Hilbert spaces of functions of two variables. In this talk, based on [1], we review an abstract scheme allowing to construct such approximations, accompanied by tight error estimates, for a wide class of asymptotically degenerating operators and their spectra. In particular, the above key example of high-contrast PDE models displays surprising links to signal processing via a novel L^2-isometric two-scale interpolation operator of Whittaker–Shannon type.

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Fujita-Type Critical Exponents for the Semilinear Parabolic Problems

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In the work, we discuss critical exponents in the sense of Fujita for semilinear parabolic equations. The talk also highlights recent advances in this direction, including our own contributions [1, 2]. The final part presents possible further developments of the problems under study.

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The Hilbert Matrix and Its Generalizations

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I discuss a new approach to the study of the classical Hilbert matrix that uses the (generalized) Mehler-Fock transform to describe its spectral properties (see [2, 3]) and its generalization as an operator acting on Bergman spaces (see [1]). I will also mention some open problems related to Hilbert matrix operators acting on Bergman spaces.

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Contributed Talks (CT)

Section: Differential Equations and Applications

Pseudo-Oscillation Equation of Heat Transfer in Thin Plates

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We consider the mixed boundary value problem of pseudo-oscillation for the heat transfer equation in a thin layer with a mid-hypersurface S in $\mathbb{R}^3$ and a boundary. The main objective is to track what happens in Γ-limit sense when the thickness of the layer converges to zero. For this, we apply the variational formulation and the calculus of Günter's tangential differential operators on a hypersurface and layers. These allow for a global representation of basic differential operators and their corresponding boundary value problems in terms of the standard Euclidean coordinates of the ambient space $\mathbb{R}^3$. The limit Dirichlet BVP for the Laplace–Beltrami equation on the surface is described explicitly. We show how the Neumann boundary conditions of the initial BVP converge in the Γ-limit sense to this BVP. In particular, the weak derivative of the Neumann condition from the initial BVP migrates into the right-hand side of the limit equation.

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Dynamic Nonclassical Transmission Problems of Generalized Thermo-Electro-Mag-Neto-Elasticity

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We investigate the solvability, asymptotic analysis and regularity results of solutions to three-dimensional dynamic nonclassical problems of interaction between thermo-elastic and generalized thermo-electro-magneto-elastic homogeneous anisotropic bodies with a crack at the interface. The considered generalized thermo-electro-magneto-elasticity model is based on the Green–Lindsay theory. Unlike classical theories of thermo-elasticity, heat propagation in this model occurs with a finite speed.

Using the Laplace transform, potential theory and the method of pseudodifferential equations on a manifold with a boundary based on the Wiener–Hopf factorization method, existence and uniqueness theorems are proved. Asymptotics of solutions near the edge of an interface crack and near the lines where the boundary conditions of different types meet are obtained. Based on the asymptotic analysis, we establish almost optimal Hölder results for solutions.

This is joint work with David Natroshvili and Tengiz Buchukuri.

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Very Weak Solutions of the Heat Equation with Anisotropically Singular Time-Dependent Diffusivity

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In this work, we investigate the Cauchy problem for the heat equation involving a time-dependent anisotropic Laplacian. We begin by studying the classical setting, where the diffusivity is smooth, and establish the existence and uniqueness of strong solutions using well-known analytical techniques. We then proceed to the singular case, where classical solutions may not exist. To handle this, we develop a rigorous theory of **very weak solutions**, which is based on regularization techniques and convolution with Gaussian kernels.

Equation of State for a Composite at Crack Tip and Displacement Discontinuity Intensity Factor

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The theory of cracks in linear fracture mechanics, without considering the material state near the crack tip—where linear elasticity laws fail—becomes ineffective in practice. Indeed, the value of the surface energy calculated using the Griffith formula for composites is ten times or more than the experimental value.

The progression of the crack is preceded by the development and branching of microcracks within a certain zone near its tip, which utilises the stored elastic energy. The properties of the material in this zone differ somewhat from those of the rest of the material. If the stress distribution does not align significantly with the root feature, the strength criterion for failure loses its validity. Fortunately, it remains possible to attribute a criterion meaning to a certain increment of the displacement jump on a crack. For instance, under the crack opening displacement (COD) condition, crack growth begins when the crack edges at the mouth reach a critical opening. Therefore, Prof. Roland Duduchava's pioneering works on the rigorous research of displacement discontinuity fields on cracks in anisotropic materials are of fundamental importance to the fracture mechanics of composites [1,2].

The report presents equations of state in the fracture process zone near a crack in polymer fiber composites, which consider the directional nature of failure and the principle of minimality of destructive forces [3]:

$$\max_{\alpha}\left[\sigma_{nn}(\alpha)+\sqrt{[m_t(\alpha_\perp)\sigma_{nt}(\alpha)]^2+[m_l(S_{21},S_{22},\alpha_\perp)\sigma_{nl}(\alpha)]^2}\right](1-\tan^2(\alpha_\perp))^{-1}=S_{22}$$

$$\max_{\beta}\left[\sigma_{ij}(\beta)L_j(\beta)L_i(\beta)\right]=S_{11}$$

where $S_{11}, S_{22}, S_{21}, \alpha_\perp$ and $m_l(S_{21}, S_{22}, \alpha_\perp),\ m_t(\alpha_\perp)$ are given strength characteristics and expressions (constants), $L_j(\beta)$ are direction cosines, α and β are the possible angles of inclination of the critical planes for the matrix and fibers.

The results of evaluating the sizes and shapes of the fracture process zone and the displacement discontinuity intensity factor using Escin's Wiener–Hopf technique for cracks located along and across the fibers are presented.

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Existence and Uniqueness Theorems for One Class of Hammerstein-Type Nonlinear Integral Equations

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The class of nonlinear integral equations on the positive half-line with a monotone operator of Hammerstein type is studied. With various partial representations of the corresponding kernel and nonlinearity, this class of equations has applications in the dynamic theory of p-adic strings, in the kinetic theory of gases, in the theory of radiation transfer, and in the mathematical theory of the geographical spread of epidemic diseases. A constructive theorem for the existence of a nontrivial bounded solution is proved. The asymptotic behavior of the constructed solution at infinity is studied. We also prove a theorem for the uniqueness of a solution in the class of nonnegative, nontrivial, and bounded functions. At the end of the work, specific particular examples of the kernel and nonlinearity of this class of equations are given, which are of independent interest. The results are related to works [1–3].

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On the Behavior of Vibrations of Non-Uniform Beams

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In this talk, we consider a clamped-clamped non-uniform beam. The free transverse vibration of a beam has the following form:

$$\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2}+k(x)w(x,t)+\frac{\partial^2}{\partial x^2}\left(EJ(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right)=0,$$

where ρ is the density of the material; $A(x)$ is the cross-sectional area; $k(x)$ is the variable coefficient of foundation; and $EJ(x)$ is the bending stiffness.

The initial conditions can be described by the shape of the beam when the external force $f(x)$ acts at $t=0$ and is then immediately released:

$$w(x,0)=v(x), \qquad \left.\frac{\partial w(x,t)}{\partial t}\right|_{t=0}=0,$$

and the clamped-clamped boundary conditions are specified as

$$w(0,t)=0, \qquad \left.\frac{\partial w(x,t)}{\partial x}\right|_{x=0}=0, \quad w(\ell,t)=0, \qquad \left.\frac{\partial w(x,t)}{\partial x}\right|_{x=\ell}=0.$$

We study an analytical solution and the spectral properties of the dynamic transverse vibration of beams under these conditions.

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Application of the Method of Helical Characteristics to the Study of Multiperiodic Solutions of Lyapunov's D-Systems

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The manifold $M'=\mathbb{R}\times\mathbb{R}^m$ for the variable (τ,t) was found to be unsuitable for studying (θ,ω)-periodic solutions in [1]. However, in the present study it becomes applicable due to the method of *helical characteristics* developed in [2]–[5].

We investigate multiperiodic solutions in (τ,t) of quasi-linear vector–matrix systems

$$Dx=Ax+f(\beta,x), \qquad D=\frac{\partial}{\partial\tau}+\sum_{j=1}^{m}\frac{\partial}{\partial t_j},$$

where A is a constant matrix and $f=(f_1,\dots,f_n)$ has Taylor expansions of order ≥ 2 in x. On the manifold $\mathcal{M}=\mathbb{R}\times S_\theta$ (with t moving along a circle S_θ of length $2\pi r=\theta$), we show that after a constant nonsingular change of variables K the matrix A takes the block-diagonal form

$$K^{-1}AK=\operatorname{diag}(\Lambda,B), \qquad \Lambda=\begin{pmatrix}0 & -\lambda\\ \lambda & 0\end{pmatrix},\ \lambda>0, \quad \Re\mu\neq 0 \text{ for } \det(B-\mu I)=0,$$

which allows us to reduce the dynamics along the characteristics of D to a Lyapunov-type subsystem.

As a consequence, in a sufficiently small neighborhood of $x = 0$ there exists a multiperiodic solution $x^*(\tau, t, \mu)$ with periods (θ^*, ω) in (τ, t), analytic in the initial data $c(\mu)$, with $\theta^*|_{\mu=0} = 2\pi/\lambda$ and θ^* analytic in (η, μ) where $\eta = \beta(\xi - \tau, t)$ (see [2]–[5]).

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On the Approximate Solution of the Initial–Boundary Value Problem for Nonlinear Kirchhoff-Type Equations with Time-Dependent Material Coefficients

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In this talk, we consider a nonlinear Kirchhoff-type equation with time-dependent material constants, which broadens the scope of its applications [1]. These include, for example, thermal softening (where heating reduces stiffness), controlled stiffness in smart structures, and aging effects (such as the gradual loss of elasticity over time).

We construct a symmetric three-layer semi-discrete scheme to approximate the solution of the corresponding Cauchy problem. In this scheme, the gradient in the nonlinear term is evaluated at the middle node. This allows the approximate solution at each time step to be obtained by inverting a linear operator. We prove the local convergence of the constructed scheme and perform numerical simulations for various model problems to illustrate its effectiveness.

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The Method of Helical Characteristics and Its Application to Problems in the Theory of Multifrequency Oscillations

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The reducibility of conditionally periodic linear equations in general position was addressed in [1]. Using a linearly conditionally periodic substitution, linear systems of ordinary differential equations were reduced to systems with constant coefficients.

We consider the problem of the reducibility of a linear multiperiodic system with a differentiation operator

$$D = \frac{\partial}{\partial \tau} + \sum_{k=1}^{m} \frac{\partial}{\partial t_k}$$

along the diagonal $t = e\tau$:

$$Dx = P(\tau, t)x, \quad P(\tau + \theta, t + \omega) = P(\tau, t) \in C^{(0,e)}_{\tau,t}(\mathbb{R} \times \mathbb{R}^m),$$

in the ϵ-neighborhood $\{|t - e\tau| < \epsilon\}$, based on a multiperiodic transformation

$$x = Q(\tau, t)y, \quad Q(\tau + \theta, t + \omega) = Q(\tau, t) \in C^{(1,e)}_{\tau,t}(\mathbb{R} \times \mathbb{R}^m), \quad \det Q(\tau, t) \neq 0,$$

to a linear system

$$Dy = \Lambda(\beta(0, \tau, t))y, \quad \Lambda(\eta + \omega) = \Lambda(\eta) \in C^{(e)}_{\eta}(\mathbb{R}^m), \quad D\beta(\xi, \tau, t) = 0,$$

with β satisfying the periodicity and composition conditions described in the original formulation.

The study was organized into five subtasks:

1^0. Establishing the equivalence between the reducibility of linear ODE systems and system (1) through substitution (2) to system (3), using the fact that $\frac{dt}{d\tau} = e$ forms the characteristic system of D [2].

2^0. Determining the characteristics $t = \beta(\tau, \xi, \eta)$ possessing θ-periodicity in τ.

3^0. Constructing an analogue of Floquet theory using a monodromy matrix constant along helical characteristics.

4^0. Studying the existence of the logarithm of the monodromy matrix under isolation, positivity, periodicity, and smoothness of eigenvalues near the helix line $(\xi, \eta) = (0, 0)$, using Gershgorin's method [3] to ensure spectral enclosure and applying results from [4] to prove smoothness and periodicity. 204 5^0. Returning from (1)–(3) to (1) via $t = e\tau$, thus establishing reducibility for linear conditionally periodic systems.

In conclusion, the method of helical characteristics establishes the reducibility of multiperiodic linear systems with a diagonal differentiation operator to systems with constant diagonal coefficients in a neighborhood of the diagonal. Transitioning to the diagonal of independent variables yields the main result on reducibility of conditionally periodic linear ODE systems in general position.

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Deep Neural Network Approach for Solving a Two-Dimensional Nonlinear Model

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We consider a two-dimensional nonlinear system of partial differential equations arising in the simulation of a biological model of vein formation in plant leaves [1]. The one-dimensional case was first studied in [2], and economical difference schemes have been investigated in [3]–[5]. To solve the initial-boundary value problem for this system, we propose a numerical method based on a Deep Neural Network (DNN), implemented in Python using the Google Colab environment [6, 7]. Numerical experiments were performed using known benchmark solutions, and the accuracy of the DNN-based approximations was evaluated by comparing them to exact analytical solutions. The observed errors confirm the effectiveness of DNN in approximating complex nonlinear PDE systems. The results demonstrate that the proposed machine learning approach offers a flexible and accurate alternative to traditional numerical methods [8].

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On Solvability of Strongly Degenerate Hyperbolic Equations

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Let $0 < T < \infty$ and let $\Omega \subset \mathbf{R}^n$ be a bounded domain with a boundary $\partial\Omega \in C^2$, $Q = \Omega \times (0, T)$, and $\Sigma = \partial\Omega \times (0, T)$. We consider the initial-boundary value problem for the degenerate hyperbolic equation

$$\partial_t \Big(t^\beta \partial_t u(x,t) \Big) - \Delta u(x,t) = f(x,t) \quad \text{in} \quad Q,$$

with conditions

$$u(x,t) = 0 \quad \text{on} \quad \Sigma,$$

$$u(x,0) = 0, \quad \lim_{t \to +0} t^\beta \partial_t u(x,t) = 0 \quad \text{in} \quad \Omega.$$

Previously, in [2], the case of weak degeneracy $\beta \in (0, 1)$ was studied. In our work, we investigate the solvability of these problems in the case of strong degeneracy, where $\beta \in [1, 2]$.

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Section: Real and Complex Analysis

Matrix Spectral Factorization Algorithm with Non-Commutative Coefficients

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Spectral factorization is a mathematical tool with various practical applications in control theory, signal processing, communications, etc. It decomposes a positive (definite) scalar (or matrix) valued function S defined on the unit circle $\mathbb{T}$ in the complex plane as

$$S(t) = S_+(t)S_+^*(t),$$

where S_+ can be extended analytically inside $\mathbb{T}$, and S_+^* is the Hermitian conjugate of S_+. The matrix spectral factorization algorithm developed in [1,2] gained popularity due to its distinctive properties [3]. However, in neuroscience applications, where very large matrices must be factorized, the proposed approach remained time-consuming.

A significant recent advancement was made by careful analysis of the method's core equation, which was rewritten in a non-commutative form, allowing the coefficients to be $M \times M$ matrices. This formulation can be solved in block-matrix form, without significantly more computation time in MATLAB compared to the original form. After the Cholesky-like factorization in the first step, instead of incrementally making leading principal submatrices analytic one-by-one, the new method processes submatrices starting from the main diagonal and doubling their size sequentially: 2×2, 4×4, 8×8, 16×16, etc. This leads to an exponential reduction in computational time for large matrices [4].

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Bombieri-Type Inequalities and Equidistribution of Points

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We present a geometric approach to the Bombieri inequalities, which involve the Bombieri–Weyl norm, a natural Hilbert space norm on homogeneous polynomials.

The optimal multiplicative estimates can be viewed in terms of optimal configurations of points. On the sphere, this is related to the condition numbers of Shub and Smale. We find a simple way of packing points on the torus that is close to optimal and gives uniform bounds independent of the number of points. This report is on joint work with Ujue Etayo and Joaquim Ortega-Cerda.

Interpolation of Variable Lebesgue Spaces Over Spaces of Homogeneous Type

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We show that the Hardy–Littlewood maximal operator M is bounded on a variable Lebesgue space $L^{p(\cdot)}(X)$, $1 < p_- \leq p_+ < \infty$, over a space of homogeneous type X *if and only if*, for every $q \in (1, \infty)$, the exponent $p(\cdot)$ can be represented as the weighted harmonic mean

$$\frac{1}{p(x)} = \frac{\theta}{q} + \frac{1-\theta}{r(x)}, \quad x \in X,$$

so that M is bounded on $L^{r(\cdot)}(X)$ for all sufficiently small $\theta > 0$. This extends the analogous result by Diening, Karlovych and Shargorodsky [1] from the Euclidean setting to spaces of homogeneous type. Such results transfer properties such as compactness of linear operators from standard Lebesgue spaces to the variable ones.

The proof of our interpolation theorem builds upon the *self-improving* boundedness property of M, recently obtained by the author [2]: if M is bounded on $L^{p(\cdot)}(X)$ over a space of homogeneous type X, then it is bounded on $L^{p(\cdot)/s}(X)$ for some $s > 1$.

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Generalized Fourier-Cesàro Sums for Functions of Bounded Variation

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In this paper, we investigate the Fourier Cesàro sums with respect to general orthonormal systems, focusing on the case when the function f belongs to a class of functions with derivatives of bounded variation.

It is known from the results of Menchov [1] and Banach [2] that the summability of general orthonormal series and the summability of general Fourier series for functions from smooth function classes are essentially different problems. Banach [2] showed that even for the constant function $f(x) = 1, x \in [0, 1]$, the summability of its Fourier series is not guaranteed with respect to arbitrary orthonormal systems.

Motivated by this distinction, our main goal is to establish conditions on the orthonormal system $\{\varphi_n\}$ such that the Fourier series of any function from the class of functions with derivatives of bounded variation is Cesàro summable. In particular, we aim to identify structural properties of the orthonormal system that ensure uniform or L^p-summability of Fourier series in the Cesàro sense.

The results contribute to a deeper understanding of summability theory in the context of non-traditional orthonormal expansions, clarifying the interplay between function smoothness and the geometry of the orthonormal system.

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Section: Algebra, Topology, and Applications

Equivariant Embeddings of Trivial G-Bundles Into Complex G-Bundles Over Finite G-CW Complexes

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The talk is based on [1], where for a complex representation V of a compact Lie group G, sufficient conditions on the isotypic parts of a complex G-vector bundle E over a G-CW complex are found so that E contains $X \times V$ as a G-subbundle.

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The Braid Group, Garside Structure and Burau Representation

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In this talk, we speak about the Burau representation faithfulness of the four-string braid group B_4, which is a well-known open problem. It is equivalent to the faithfulness of the Jones representation, which itself is related to the Jones hypothesis stating that the Jones polynomial detects the unknot. I will review the Garside and dual Garside structures on the braid group and their role in the Lawrence–Krammer–Bigelow representation. Finally, I will discuss the current progress on the Burau representation faithfulness problem in collaboration with my co-authors.

Relations Between Categories of Crossed Modules of Various Algebras

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Groups, associative algebras, and Lie algebras are related by the well-known adjoint functors: the group algebra functor is left adjoint to the unit group functor, and the universal enveloping algebra functor is left adjoint to the Liezation functor. These classical facts have been recently extended to the respective categories of crossed modules [2,3].

In the non-commutative framework, when Lie algebras are replaced by Leibniz algebras, the analogous objects to associative algebras are dialgebras, introduced and studied by J.-L. Loday [4]. There is an adjunction between the categories of Leibniz algebras and dialgebras, which is analogous and related to the one between the categories of Lie and associative algebras.

We present the construction of adjoint functors between the categories of crossed modules of dialgebras and Leibniz algebras. Moreover, we extend the well-known relations between the categories of Lie, Leibniz, associative algebras, and dialgebras to the respective categories of crossed modules.

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On Factorization of Groups, Monoids and Skew Braces

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It has long been understood that the problem of determining whether a mathematical object satisfies a given property can often be reduced to representing it as arising from (usually simpler) substructures with "minimal" intersection via a suitable categorical construction (such as a product, coproduct, or other categorical operation). These substructures are generally more amenable to existing techniques, even when such techniques are not directly applicable to the original object. This type of decomposition is referred to as a factorization of the object.

In this talk, we consider the factorization problem for groups, monoids, and skew braces. We provide a complete classification of factorizations of these structures in terms of *complementary pairs of quasi-projections*.

Extremal Problems on Four Concentric Circles

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We present results on the stationary configurations of the oriented area and perimeter on the reduced configuration space of four concentric circles in the plane. More precisely, we describe all critical configurations of the oriented area, which, in particular, yield explicit formulae for its extremal values. Our results generalize analogous results of E. Gutkin for three concentric circles [1].

Theorem 1. The oriented area is an exact Morse function on the reduced configuration space of four concentric circles in the Euclidean plane.

All these configurations are non-degenerate, and the collection of their Morse indices is $(0,1,1,1,2,2,2,3)$. We also give explicit formulas for the extremal values of the oriented area.

Theorem 2. The perimeter is generically a Morse function on the reduced configuration space of four concentric circles in the Euclidean plane. It always has eight critical points represented by the aligned configurations, but may also have convex or self-intersecting critical quadrilaterals.

Our results imply, in particular, a criterion of existence of convex critical configurations and suggest several open problems and plausible conjectures. More precisely, the criterion is positivity of the following expression:

$$E = (Q - r_1r_2r_3)(Q - r_1r_2r_4)(Q - r_2r_3r_4)(Q - r_1r_3r_4),$$

where r_i are the radii of the given circles and

$$Q = \frac{1}{2}(r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4).$$

Since this expression is negative, for example, for radii $(1,2,3,12)$, we conclude that in this case, there is no convex critical configuration of perimeter. The proofs are based on the use of the so-called Snellius law [2].

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Homotopy in the Geometry of Monoids

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The geometry of monoids is a term used to describe new types of algebraic geometry where monoids play a central role. Examples include F_1 geometry, tropical geometry, toric varieties and many others. Much like (ring-)schemes, monoid schemes play a crucial part in all of these.

We can obtain classical varieties from monoid schemes via realisations. There are two particularly noteworthy such processes: locally, one is based on monoid rings $K[M]$, the other is based on Hom(M,K). Relevant to our talk is the latter, which is denoted by KSpec(M) or KX for a monoid scheme X. This construction generalises toric varieties.

We will focus on the cases $K = \mathbb{C}$ and $\mathbb{R}$. For $\mathbb{R}$, Professor Holger Brenner and I developed an explicit and algorithmic way of calculating its fundamental groupoid. Much of it can also be generalised to $\mathbb{C}$ and, if time permits, we will mention this example as well. We will also talk about their homologies.

Acknowledgment. Parts of the results discussed in this talk were developed in joint work with Professor Brenner from the University of Osnabrück.

An Isometry Theorem for Persistent Homology of Circle-Valued Functions

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Circle-valued functions generalise real-valued ones by taking values on a circle rather than a line, making them well-suited for data with periodic or directional structure.

In persistent homology, such functions lead to circle-valued persistence modules, which can be viewed as representations of a cyclic zig-zag quiver of type $\tilde{A}_n$. Burghelea and Dey classified their indecomposable representations as barcodes and Jordan blocks and proposed an algorithm to compute them.

Stability of persistent homology relies on the interleaving distance, which has been extended to zig-zag and poset representations using tools such as the Auslander–Reiten translate.

Our main result is a stable interleaving distance for circle-valued persistence modules. We also introduce a computer-friendly geometric model from representation theory to encode their invariants and define a matching distance via this model, which we show coincides with the interleaving distance.

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On k-Invariants of Crossed Modules Related to Dihedral Groups

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We present some recent results obtained in the joint work with Guram Donadze. We give an explicit computation of the k-invariant of crossed modules of the form $G \to \mathrm{Aut}(G)$, where G is the dihedral group.

On Projective Class Groups of Crossed Group Rings

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Let K be the quotient field of a Dedekind ring R, and $R[\pi, \rho]$, $K[\pi, \rho]$ crossed products. We will consider finitely generated projective $R[\pi, \rho]$-modules such that $K \otimes P$ is $K[\pi, \rho]$-free. Let $\mathbf{P}_0$ be the category of such projectives and let $P_0(R[\pi, \rho])$ be the Grothendieck group of $\mathbf{P}_0$. Let $P_0^{'}(R[\pi, \rho])$ be the subgroup of $P_0(R[\pi, \rho])$ generated by all $[F]$ with F free over $R[\pi, \rho]$.

The special projective class group is defined to be

$$C_0(R[\pi, \rho]) = P_0(R[\pi, \rho])/P_0^{'}(R[\pi, \rho]).$$

Let M be some class of subgroups of a finite group π, F some kind of algebraic K-functors, and $F^M(R[\pi, \rho])$ the sum of the images of the maps $F(R[\pi', \rho]) \to F(R[\pi, \rho])$ induced by $i : \pi' \subset \pi$, $\pi' \in M$. We investigate some properties of the exponents of $F^M(R[\pi,\rho])$ in $F(R[\pi,\rho])$ for cyclic $M = C$, elementary $M = E$, and hyperelementary $M = H$ subgroups of π, under natural restrictions on R and π.

Theorem 1. Suppose $\mathrm{char}(R)$ is prime to $(\pi : 1)$. If $G_M(K[\pi,\rho])$ has exponent k in $G(K[\pi,\rho])$, then $C_0^M(R[\pi,\rho])$ has exponent k in $C_0(R[\pi,\rho])$.

Theorem 2. Suppose $\mathrm{char}(R)$ is prime to $(\pi : 1)$. Then

(a) $C_0^C(R[\pi,\rho])$ has exponent n in $C_0(R[\pi,\rho])$;

(b) $C_0^E(R[\pi,\rho])$ has exponent d in $C_0(R[\pi,\rho])$;

(c) $C_0^H(R[\pi,\rho]) = C_0(R[\pi,\rho])$.

Another Characterization of Left Hereditary Left Perfect Right Coherent Rings

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The Chase's characterization of left perfect right coherent rings is well known [1]. It asserts that a ring is such if and only if the direct product of an arbitrary small family of projective left modules is projective. Applying this theorem, as well as a purely categorical result—a certain form of the well-known epireflective subcategory theorem—we prove that *a left hereditary ring is left perfect and right coherent if and only if an arbitrary left module over it can be decomposed into the direct sum of a projective left module and a stable left module (i.e., a left module that has no nonzero projective direct summands). In that case, this decomposition is unique up to isomorphism and also is functorial.*

Employing the same categorical approach, we give a new proof of He's structure theorem for left modules over left hereditary left Noetherian rings [2]. The results of the present work can be found in the author's preprint [3].

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[3] D. Zangurashvili. On stable-projective and injective-costable decompositions of modules. `https://arxiv.org/abs/2302.11202`.

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If you require a **taxi**, please be aware that **taxis** in Tbilisi are typically unmetered, and as a result, tourists can occasionally fall victim to overcharging. This makes hailing taxis from the street a somewhat risky endeavour. We highly recommend downloading a ride-hailing app such as **Bolt** and using it to book your taxis in Tbilisi. A journey between the airport and the city centre, or vice versa, when booked through **Bolt**, should commonly not exceed ***15 Euros*** in cost.

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For lunch, we recommend the restaurants **"Samikitno-Matchakhela"** and **"Khinkali Factory"**, both located near the **University of Georgia (UG)**. Their address is: **77 Merab Kostava St., Tbilisi 0171**. If you prefer to have food delivered to your location, Tbilisi offers several international delivery apps, including **Glovo**, **Wolt**, and **Bolt Food**. In addition, during the summer season, a local cafeteria may be available on the second floor of the university.

Like many countries, most restaurants, supermarkets, and other establishments in our region accept **credit** or **debit card** payments (**Visa**, **MasterCard**, or **Amex**). However, if you prefer to use **cash**, we recommend exchanging currency on **Pekini Ave.**, which is conveniently located near the **conference venue**. Please be aware that exchanging currency at **banks**, **airports**, and in **touristic zones** (such as the Liberty Square neighbourhood) might not provide favourable exchange rates for you.

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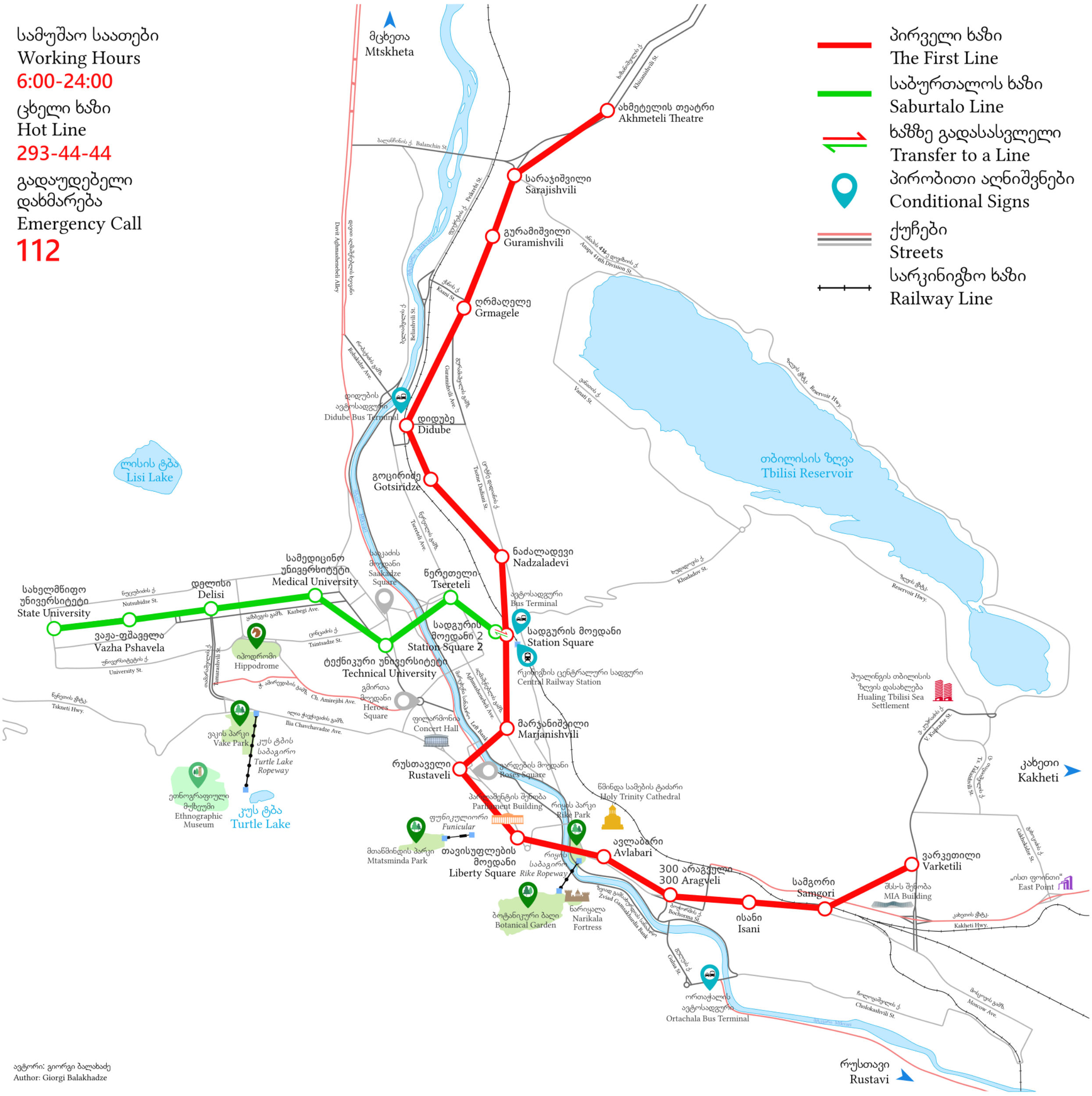

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